HANSER

Polymer Processing

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Modeling and Simulation

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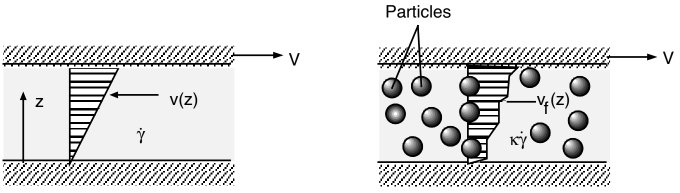

Figure 2.41: Schematic diagram of strain rate increase in a filled system.

processing. Numerous models have been proposed to estimate the viscosity of filled liquids [3, 15, 23, 25, 26]. Most models proposed are a power series of the form

$$\frac{\eta_f}{\eta} = 1 + a_1\phi + a_2\phi^2 + a_3\phi^3 + ... \tag{2.71}$$

The linear term in eqn. (2.71) represents the narrowing of the flow passage caused by the filler that is passively entrained by the fluid and sustains no deformation as shown in Fig. 2.41.

For instance, Einstein's model, which only includes the linear term with $a_1 = 2.5$, was derived based on a viscous dissipation balance. The quadratic term in the equation represents the first-order effects of interaction between the filler particles. Geisbüsch suggested a model with a yield stress and, where the strain rate of the melt increases by a factor κ as

$$\eta_f = \frac{\tau_0}{\dot{\gamma}} + \kappa\eta_0(\kappa\dot{\gamma}) \tag{2.72}$$

For high deformation stresses, which are typical in polymer processing, the yield stress in the filled polymer melt can be neglected. Figure 2.42 compares Geisbüsch's experimental data to eqn. (2.71) using the coefficients derived by Guth [25]. The data and Guth's model seem to agree well. A comprehensive survey on particulate suspensions was recently given by Gupta [25], and on short-fiber suspensions by Milliken and Powell [52].

2.3.3 Viscoelastic Constitutive Models

Viscoelasticity has already been introduced in Chapter 1, based on linear viscoelasticity. However, in polymer processing large deformations are imposed on the material, requiring the use of non-linear viscoelastic models. There are two types of general non-linear viscoelastic flow models: the differential type and the integral type.

Differential Viscoelastic Models. Differential models have traditionally been the choice for describing the viscoelastic behavior of polymers when simulating complex flow systems. Many differential viscoelastic models can be described by the general form

$$Y\boldsymbol{\tau} + \lambda_1\boldsymbol{\tau}_{(1)} + \lambda_2\{\dot{\boldsymbol{\gamma}}\cdot\boldsymbol{\tau} + \boldsymbol{\tau}\cdot\dot{\boldsymbol{\gamma}}\} + \lambda_3\{\boldsymbol{\tau}\cdot\boldsymbol{\tau}\} = \eta_0\left[\dot{\boldsymbol{\gamma}} + \lambda_4\boldsymbol{\gamma}_{(2)}\right] \tag{2.73}$$

where $\boldsymbol{\tau}_{(1)}$ is the first contravariant convected time derivative of the deviatoric stress tensor and represents rates of change with respect to a convected coordinate system that moves and deforms with the fluid. The convected derivative of the deviatoric stress tensor is defined as

$$\boldsymbol{\tau}_{(1)} = \frac{D\boldsymbol{\tau}}{Dt} - \left[(\nabla\mathbf{u})^T\cdot\boldsymbol{\tau} + \boldsymbol{\tau}\cdot(\nabla\mathbf{u})\right] \tag{2.74}$$

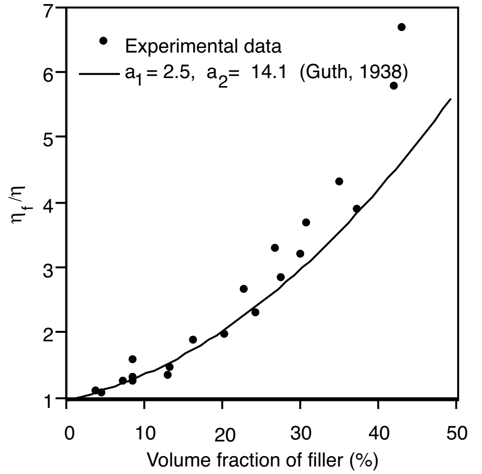

Figure 2.42: Viscosity increase as a function of volume fraction of filler for polystyrene and low density polyethylene containing spherical glass particles with diameters ranging between $36\mu m$ and $99.8\mu m$.

Table 2.7: Definition of Constants in eqn. (2.73)

Constitutive model	Y	λ_1	λ_2	λ_3	λ_4
Generalized Newtonian	1	0	0	0	0
Upper convected Maxwell	1	λ_1	0	0	0
Convected Jeffreys	1	λ_1	0	0	λ_4
White-Metzner	1	$\lambda_1(\dot{\gamma})$	0	0	0
Phan-Thien Tanner-1	$e^{(-\epsilon(\lambda/\eta_0)\mathrm{tr}\boldsymbol{\tau})}$	λ	$\frac{1}{2}\xi\lambda$	0	0
Phan-Thien Tanner-2	$1-\epsilon\,(\lambda/\eta_0)\,\mathrm{tr}\boldsymbol{\tau}$	λ	$\frac{1}{2}\xi\lambda$	0	0
Giesekus	1	λ_1	0	$-\left(\alpha\lambda_1/\eta_0\right)$	0

The constants in eqn. (2.73) are defined in Table 2.7 for various viscoelastic models commonly used to simulate polymer flows.

A review by Bird and Wiest [6] gives a more complete list of existing viscoelastic models. The upper convective model and the White-Metzner model are very similar with the exception that the White-Metzner model incorporates the strain rate effects of the relaxation time and the viscosity. Both models provide a first order approximation to flows, in which shear rate dependence and memory effects are important. However, both models predict zero second normal stress coefficients. The Giesekus model is molecular-based, non-linear in nature and describes the power law region for viscosity and both normal stress coefficients. The Phan-Thien Tanner models are based on network theory and give non-linear stresses. Both the Giesekus and Phan-Thien Tanner models have been successfully used to model complex flows.

■ **EXAMPLE 2.2.**

Shearing flows of the convected Jeffreys model. The convected Jeffreys model [6] or Oldroyd's B-fluid [54] is given by,

$$\boldsymbol{\tau} + \lambda_1 \boldsymbol{\tau}_{(1)} = \eta_0 \left[\dot{\boldsymbol{\gamma}} + \lambda_2 \boldsymbol{\gamma}_{(2)} \right] \tag{2.75}$$

Here we have three parameters: η_0 the zero-shear-rate viscosity, λ_1 the relaxation time and λ_2 the retardation time. In the case of $\lambda_2 = 0$ the model reduces to the convected Maxwell model, for $\lambda_1 = 0$ the model simplifies to a second-order fluid with a vanishing second normal stress coefficient [6], and for $\lambda_1 = \lambda_2$ the model reduces to a Newtonian fluid with viscosity η_0. If we impose a shear flow,

$$\frac{\partial u_x}{\partial y} = \dot{\gamma}_{yx}(t) \tag{2.76}$$

the constitutive equation (eqn. (2.75)) will be in tensor form (Table 2.8),

$$\begin{bmatrix} \tau_{xx} & \tau_{yx} & 0 \\ \tau_{yx} & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} + \lambda_1 \frac{d}{dt} \begin{bmatrix} \tau_{xx} & \tau_{yx} & 0 \\ \tau_{yx} & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} - \lambda_1 \dot{\gamma}_{yx} \begin{bmatrix} 2\tau_{yx} & \tau_{yy} & 0 \\ \tau_{yy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$= \eta_0 \left\{ \dot{\gamma}_{yx} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \lambda_2 \frac{d\dot{\gamma}_{yx}}{dt} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - 2\lambda_2 \dot{\gamma}_{yx}^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \tag{2.77}$$

From this equations we can obtain the following set of partial differential equations,

$$\begin{aligned} \left(1 + \lambda_1 \frac{d}{dt}\right) \tau_{xx} - 2\tau_{yx}\lambda_1\dot{\gamma}_{yx}(t) &= -\,2\eta_0\lambda_2\dot{\gamma}_{yx}^2(t) \\ \left(1 + \lambda_1 \frac{d}{dt}\right) \tau_{yy} &= 0 \\ \left(1 + \lambda_1 \frac{d}{dt}\right) \tau_{zz} &= 0 \\ \left(1 + \lambda_1 \frac{d}{dt}\right) \tau_{yx} - \tau_{yy}\lambda_1\dot{\gamma}_{yx}(t) &= \eta_0 \left(1 + \lambda_2 \frac{d}{dt}\right) \dot{\gamma}_{yx}(t) \end{aligned} \tag{2.78}$$

which indicates that the normal stresses τ_{yy} and τ_{zz} are zero for any time-dependent shearing flow.

For *steady shear flow* these differential equations are simplified to give,

$$\begin{aligned}\tau_{yx} =&\eta_0\dot{\gamma}_{yx}\\ \tau_{xx}-\tau_{yy} =&2\eta_0(\lambda_1-\lambda_2)\dot{\gamma}_{yx}^2\\ \tau_{yy}-\tau_{zz} =&0\end{aligned} \tag{2.79}$$

and we obtain the following viscometric functions,

$$\begin{aligned}\eta =&\eta_0\\ \Psi_1 =&2\eta_0(\lambda_1-\lambda_2)\\ \Psi_2 =&0\end{aligned} \tag{2.80}$$

Indicating that the convected Jeffreys model gives a constant viscosity and first normal stress coefficient, while the second normal stress coefficient is zero.

For a *small amplitude oscillatory shearing flow*, the strain is defined as,

$$\gamma_{yx}(t) = \int_0^t \dot{\gamma}_0 \cos wt' dt' = \gamma_0 \sin wt \tag{2.81}$$

where $\gamma_0 = \dot{\gamma}_0/w$. The differential equation for the shear stress will be,

$$\left(1+\frac{d}{dt}\right)\tau_{yx} = \eta_0\gamma_0 w\left(\cos wt - \lambda_2 w \sin wt\right) \tag{2.82}$$

Seeking a steady periodic solution, the right hand side suggest that the solution should be [6],

$$\tau_{yx} = A\cos wt + B\sin wt \tag{2.83}$$

which, after replacing it into the original equation, we obtain,

$$\begin{aligned}A &= \eta_0\left(\frac{1+\lambda_1\lambda_2 w^2}{1+\lambda_1^2 w^2}\right)\gamma_0 w = \eta'(w)\gamma_0 w\\ B &= \eta_0\left(\frac{(\lambda_1-\lambda_2)\gamma_0 w^2}{1+\lambda_1^2 w^2}\right)\gamma_0 w = \eta''(w)\gamma_0 w\end{aligned} \tag{2.84}$$

■ EXAMPLE 2.3.

Steady shearfree flow for the White-Metzner model. This model is a nonlinear model which modifies the convected Maxwell model by including the dependence on $\dot{\gamma}$ in the viscosity, i.e.,

$$\boldsymbol{\tau} + \lambda_1(\dot{\gamma})\boldsymbol{\tau}_{(1)} = \eta(\dot{\gamma})\dot{\boldsymbol{\gamma}} \tag{2.85}$$

where $\dot{\gamma} = \sqrt{1/2\dot{\boldsymbol{\gamma}}:\dot{\boldsymbol{\gamma}}}$. For a shearfree flow we have that (Table 2.9),

$$\dot{\boldsymbol{\gamma}} = \dot{\epsilon}(t)\begin{bmatrix}-(1+b) & 0 & 0\\ 0 & (1-b) & 0\\ 0 & 0 & 2\end{bmatrix} \tag{2.86}$$

Table 2.8: Shearing Flow Tensors $\mathbf{u} = (\dot{\gamma}_{yx}(t)y, 0, 0)$ [6]

$\nabla \mathbf{u}$	$\dot{\gamma}_{yx}(t) \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$\dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma}^{(\mathbf{1})} = \boldsymbol{\gamma}_{(\mathbf{1})}$	$\dot{\gamma}_{yx}(t) \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$\boldsymbol{\gamma}^{(2)}$	$\dfrac{\partial \dot{\gamma}_{yx}}{\partial t} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \dot{\gamma}_{yx}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$\boldsymbol{\gamma}_{(2)}$	$\dfrac{\partial \dot{\gamma}_{yx}}{\partial t} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \dot{\gamma}_{yx}^2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$\boldsymbol{\tau} = \boldsymbol{\tau}^{(\mathbf{0})} = \boldsymbol{\tau}_{(\mathbf{0})}$	$\begin{bmatrix} \tau_{xx} & \tau_{yx} & 0 \\ \tau_{yx} & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix}$
$\boldsymbol{\tau}^{(1)}$	$\dfrac{\partial}{\partial t} \begin{bmatrix} \tau_{xx} & \tau_{yx} & 0 \\ \tau_{yx} & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} + \dot{\gamma}_{yx} \begin{bmatrix} 0 & \tau_{xx} & 0 \\ \tau_{xx} & 2\tau_{yx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$\boldsymbol{\tau}_{(1)}$	$\dfrac{\partial}{\partial t} \begin{bmatrix} \tau_{xx} & \tau_{yx} & 0 \\ \tau_{yx} & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} - \dot{\gamma}_{yx} \begin{bmatrix} 2\tau_{yx} & \tau_{yy} & 0 \\ \tau_{yy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

for a steady flow $\dot{\epsilon}(t) = \dot{\epsilon}_0$ and

$$\dot{\boldsymbol{\gamma}} \cdot \dot{\boldsymbol{\gamma}} = \dot{\epsilon}_0^2 \begin{bmatrix} (1+b)^2 & 0 & 0 \\ 0 & (1-b)^2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \tag{2.87}$$

and we have

$$\frac{1}{2}\dot{\boldsymbol{\gamma}} : \dot{\boldsymbol{\gamma}} = \frac{1}{2}\mathrm{tr}\dot{\boldsymbol{\gamma}} \cdot \dot{\boldsymbol{\gamma}} = \sqrt{3+b^2}|\dot{\epsilon}_0| \tag{2.88}$$

Here $0 \leq b \leq 1$ and $\dot{\epsilon}$ is the elongation rate. Several special shearfree flows are obtained for particular choices of b, i.e.,

$b = 0$ and $\dot{\epsilon} > 0$ Elongational flow
$b = 0$ and $\dot{\epsilon} < 0$ Biaxial stretching flow
$b = 1$ Plannar elongational flow

The tensor form of the constitutive equation is,

$$\begin{aligned} \begin{bmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} - \lambda_1(\dot{\gamma}) \begin{bmatrix} -(1+b)\tau_{xx} & 0 & 0 \\ 0 & -(1-b)\tau_{yy} & 0 \\ 0 & 0 & 2\tau_{zz} \end{bmatrix} \dot{\epsilon}_0 \\ = \eta(\dot{\gamma}) \begin{bmatrix} -(1+b) & 0 & 0 \\ 0 & -(1-b) & 0 \\ 0 & 0 & 2 \end{bmatrix} \dot{\epsilon}_0 \end{aligned} \tag{2.89}$$

which will give us the following differential equations,

$$\begin{aligned} \tau_{xx}\left[1 + (1+b)\left(\frac{\eta}{G}\dot{\epsilon}_0\right)\dot{\epsilon}_0\right] &= -(1+b)\eta\dot{\epsilon}_0 \\ \tau_{yy}\left[1 + (1-b)\left(\frac{\eta}{G}\dot{\epsilon}_0\right)\dot{\epsilon}_0\right] &= -(1-b)\eta\dot{\epsilon}_0 \\ \tau_{zz}\left[1 - 2\left(\frac{\eta}{G}\dot{\epsilon}_0\right)\dot{\epsilon}_0\right] &= 2\eta\dot{\epsilon}_0 \end{aligned} \tag{2.90}$$

From these equations we get the elongational viscosities as [6],

$$\begin{aligned} \bar{\eta}_1 = \tau_{zz} - \tau_{xx} &= \frac{(3+b)\eta(\dot{\gamma})\dot{\epsilon}_0}{[1+(1+b)(\eta/G)\dot{\epsilon}_0]\,[1-2(\eta/G)\dot{\epsilon}_0]} \\ \bar{\eta}_2 = \tau_{yy} - \tau_{xx} &= \frac{2b\eta(\dot{\gamma})\dot{\epsilon}_0}{[1+(1+b)(\eta/G)\dot{\epsilon}_0]\,[1+(1-b)(\eta/G)\dot{\epsilon}_0]} \end{aligned} \tag{2.91}$$

Integral viscoelastic models. Integral models with a memory function have been widely used to describe the viscoelastic behavior of polymers and to interpret their rheological measurements [37, 41, 43]. In general one can write the single integral model as

$$\boldsymbol{\tau} = \int_{-\infty}^{t} M(t-t')\mathbf{S}(t')dt' \tag{2.92}$$

Table 2.9: Shearfree Flow Tensors $\mathbf{u} = (-1/2(1+b)\dot{\epsilon}(t)x, -1/2(1+b)\dot{\epsilon}(t)y, \dot{\epsilon}(t)z)$ [6]

$\nabla \mathbf{u}$	$\dot{\epsilon}(t)\begin{bmatrix} -1/2(1+b) & 0 & 0 \\ 0 & -1/2(1-b) & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$\dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma}^{(\mathbf{1})} = \boldsymbol{\gamma}_{(\mathbf{1})}$	$\dot{\epsilon}(t)\begin{bmatrix} -(1+b) & 0 & 0 \\ 0 & -(1-b) & 0 \\ 0 & 0 & 2 \end{bmatrix}$
$\boldsymbol{\gamma}^{(2)}$	$\frac{\partial \dot{\epsilon}}{\partial t}\begin{bmatrix} -(1+b) & 0 & 0 \\ 0 & -(1-b) & 0 \\ 0 & 0 & 2 \end{bmatrix} + \dot{\epsilon}^2\begin{bmatrix} (1+b)^2 & 0 & 0 \\ 0 & (1-b)^2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
$\boldsymbol{\gamma}_{(2)}$	$\frac{\partial \dot{\epsilon}}{\partial t}\begin{bmatrix} -(1+b) & 0 & 0 \\ 0 & -(1-b) & 0 \\ 0 & 0 & 2 \end{bmatrix} - \dot{\epsilon}^2\begin{bmatrix} (1+b)^2 & 0 & 0 \\ 0 & (1-b)^2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
$\boldsymbol{\tau} = \boldsymbol{\tau}^{(\mathbf{0})} = \boldsymbol{\tau}_{(\mathbf{0})}$	$\begin{bmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix}$
$\boldsymbol{\tau}^{(1)}$	$\frac{\partial}{\partial t}\begin{bmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} + \dot{\epsilon}\begin{bmatrix} -(1+b)\tau_{xx} & 0 & 0 \\ 0 & -(1-b)\tau_{yy} & 0 \\ 0 & 0 & 2\tau_{zz} \end{bmatrix}$
$\boldsymbol{\tau}_{(1)}$	$\frac{\partial}{\partial t}\begin{bmatrix} \tau_{xx} & 0 & 0 \\ 0 & \tau_{yy} & 0 \\ 0 & 0 & \tau_{zz} \end{bmatrix} - \dot{\epsilon}\begin{bmatrix} -(1+b)\tau_{xx} & 0 & 0 \\ 0 & -(1-b)\tau_{yy} & 0 \\ 0 & 0 & 2\tau_{zz} \end{bmatrix}$

Table 2.10: Definition of Constants in eqn. (2.73)

Constitutive model	ϕ_1	ϕ_2
Lodge rubber-like liquid	1	0
K-BKZ[a]	$\dfrac{\partial W}{\partial I_1}$	$\dfrac{\partial W}{\partial I_2}$
Wagner[b]	$e^{\beta}\sqrt{\alpha I_1 + (1-\alpha)I_2 - 3}$	0
Papanastasiou-Scriven-Macosko[c]	$\dfrac{\alpha}{(\alpha-3)+\beta I_1+(1-\beta)I_2}$	0

[a] $W(I_1, I_2)$ represents a potential function which can be derived from empiricisms or molecular theory; [b] Wagner's model is a special form of the K-BKZ model; [c] The Papanastasiou-Scriven-Macosko model is also a special form of the K-BKZ model

where $M(t-t')$ is a memory function and $\mathbf{S}(t')$ a deformation dependent tensor defined by

$$\mathbf{S}(t') = \phi_1(I_1, I_2)\boldsymbol{\gamma}_{[0]} + \phi_2(I_1, I_2)\boldsymbol{\gamma}^{[0]} \tag{2.93}$$

where I_1 and I_2 are the first invariants of the Cauchy and Finger strain tensors, respectively.

Table 2.10 [4, 36, 42, 69] defines the constants ϕ_1 and ϕ_2 for various models. In eqn. (2.93), $\boldsymbol{\gamma}_{[0]}$ and $\boldsymbol{\gamma}^{[0]}$ are the finite strain tensors given by

$$\begin{aligned} \boldsymbol{\gamma}_{[0]} &= \boldsymbol{\Delta}^t \cdot \boldsymbol{\Delta} - \boldsymbol{\delta} \\ \boldsymbol{\gamma}^{[0]} &= \boldsymbol{\delta} - \mathbf{E} \cdot \mathbf{E}^t \end{aligned} \tag{2.94}$$

The terms Δ_{ij} and E_{ij} are displacement gradient tensors[4] defined by

$$\begin{aligned} \Delta_{ij} &= \frac{\partial x_i'(x, t, t')}{\partial x_j} \\ E_{ij} &= \frac{\partial x_i(x', t', t)}{\partial x_j'} \end{aligned} \tag{2.95}$$

where the components Δ_{ij} measure the displacement of a particle at past time t' relative to its position at present time t, and the terms E_{ij} measure the material displacements at time t relative to the positions at time t'.

A memory function $M(t-t')$, which is often applied and which leads to commonly used constitutive equations, is written as

$$M(t-t') = \sum_{k=1}^{n} \frac{\eta_k}{\lambda_k^2} e^{(-\frac{t-t'}{\lambda_k})} \tag{2.96}$$

[4] Another combination of the displacement gradient tensors which are often used are the Cauchy strain tensor and the Finger strain tensor defined by $\mathbf{B^{-1}} = \boldsymbol{\Delta}^t\boldsymbol{\Delta}$ and $\mathbf{B} = \mathbf{E}\mathbf{E^t}$, respectively.

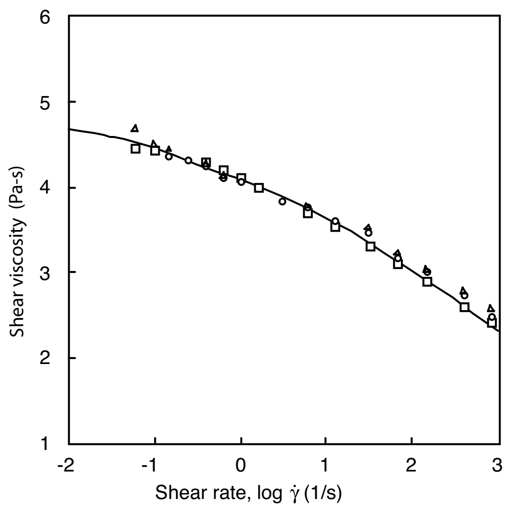

Figure 2.43: Measured and predicted shear viscosity for various high density poly-ethylene resins at 170^{o}C.

where λ_k and η_k are relaxation times and viscosity coefficients at the reference temperature T_{ref}, respectively. Once a memory function has been specified one can calculate several material functions using [6]

$$\begin{aligned} \eta(\dot{\gamma}) &= \int_0^\infty M(s)s(\phi_1 + \phi_2)ds \\ \psi_1(\dot{\gamma}) &= \int_0^\infty M(s)s^2(\phi_1 + \phi_2)ds \\ \psi_2(\dot{\gamma}) &= \int_0^\infty M(s)s^2(\phi_2)ds \end{aligned} \tag{2.97}$$

For example, Figs. 2.43 and 2.44 present the measured [55] viscosity and first normal stress difference data, respectively, for three blow molding grade high density polyethylenes along with a fit obtained from the Papanastasiou-Scriven-Macosko [59] form of the K-BKZ equation. A memory function with a relaxation spectrum of 8 relaxation times was used.

The coefficients used to fit the data are summarized in Table 2.11 [43]. The viscosity and first normal stress coefficient data presented in Figs. 2.30 and 2.31 where fitted with the Wagner form of the K-BKZ equation [41].

■ EXAMPLE 2.4.

Shear flow for a Lodge rubber-liquid. If we consider the flow field,

$$\begin{aligned} u_x &= \dot{\gamma}_{yx}(t)y \\ u_y &= u_z = 0 \end{aligned} \tag{2.98}$$

and we are seeking an expression of the stress tensor of a Lodge rubber-liquid, we start from the integral form of the stress tensor

$$\boldsymbol{\tau} = \int_{-\infty}^{t} M(t-t')\mathbf{S}(t')dt' \tag{2.99}$$