

# HANSER

Sample Pages

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Measurement Process Qualification

Gage Acceptance and Measurement Uncertainty According to Current  
Standards

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### 8.2.7 Standard Uncertainty Caused by the Operator Influence $u_{AV}$

Since every inspector handles a measuring device differently, the resulting measurement deviation is significant. In order to assess the operator influence, several operators (e.g. 3) may take several repeat measurements on several objects (e.g. 10), as described in chapter 8.2.6.

#### Example

The standard uncertainty caused by the appraiser variation  $u_{AV}$  for the series of measurements in Table 8-4 amounts to  $u_{AV} \approx 0.1\mu\text{m}$  according to the ANOVA method (see Figure 8-7).

### 8.2.8 Standard Uncertainty Caused by the Test Object $u_{OBJ}$

The variation of the test objects (part variation) is another influencing factor affecting the measurement process. In repeat measurements, form and shape deviations lead to a measurement deviation at the same test object. This deviation must be considered as standard uncertainty  $u_{pa}$ . Depending on the material/properties of the test objects, the properties might even change over time (elasticity, viscosity, etc.). In order to determine the influence of the test objects, an inspector takes several repeat measurements (at least 20) from one test object. The standard deviation calculated from the series of measurements corresponds to the wanted standard measurement uncertainty component  $u_{pa}$ .

#### Examples

1. If the figure specifies the form deviation (see Figure 8-8) it will be monitored during the production process in order that no parts show a form deviation exceeding the one given in the figure.  $u_{OBJ}$  can be calculated from the tolerance of the figure  $TOL = 3\mu\text{m}$ :

$$u_{OBJ} = \frac{TOL}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{1}{\sqrt{3}} \approx 1.7\mu\text{m}$$

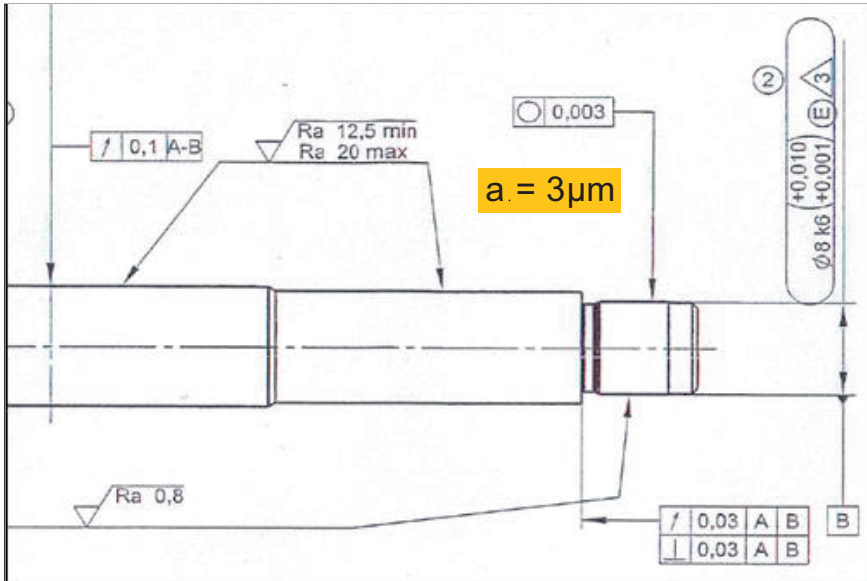


Figure 8-8: Standard uncertainty object influence from tolerance

**Note**

This calculation method leads to the greatest standard uncertainty besides the influence of the object. Particular measurements on the object (see example 2 and 3) lead to a smaller standard uncertainty.

2. If an appropriate measuring device measures the form deviation, the distance  $a = 1\ \mu m$  and the standard deviation  $s_g = 0.2$  can be taken from the record in case of this example (see Figure 8-9). This leads to the standard uncertainty of the object variation

$$u_{OBJ} = \frac{a}{\sqrt{3}} = \frac{1\ \mu m}{\sqrt{3}} = 0.577 \approx 0.6\ \mu m \text{ or}$$

$$u_{OBJ} = 2 \cdot s_g = 2 \cdot 0.2 \approx 0.4\ \mu m$$

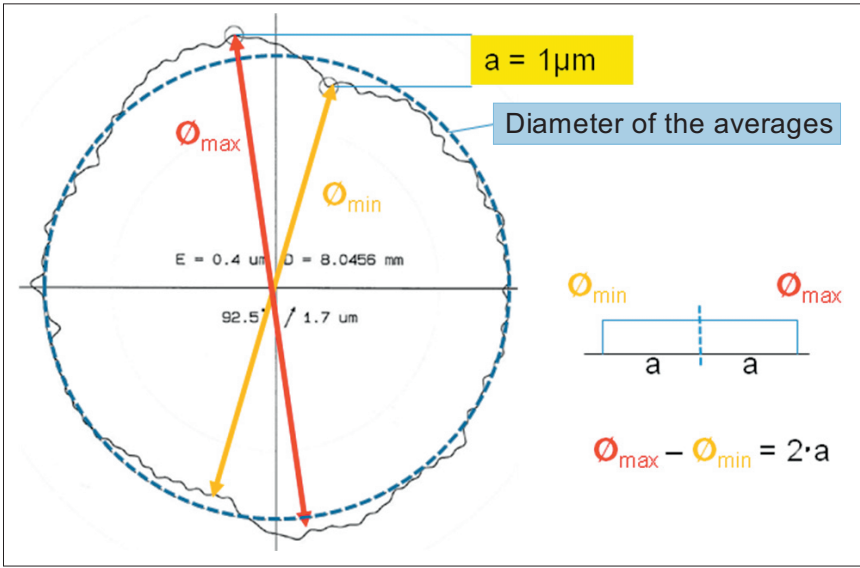


Figure 8-9: Standard uncertainty object influence from measured object variation

3. In an air-conditioned room, an inspector takes 20 repeat measurements from a flange heated to 20°C. The values listed in Table 8-5 lead to the value chart displayed in Figure 8-10.

$x_i$	$x_i$	$x_i$	$x_i$	$x_i$
5,5	7,0	7,5	6,5	7,0
7,5	6,0	7,0	5,5	6,5
6,5	6,0	6,5	6,5	5,0
8,0	7,0	7,5	8,0	7,0
8,0	8,0	5,5	7,5	6,5

Table 8-5: Repeat measurements

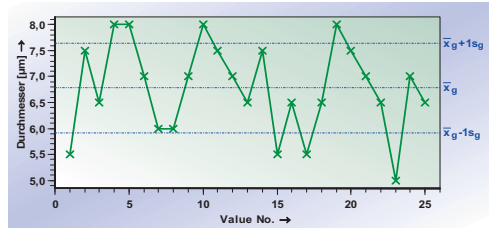


Figure 8-10: Actual value chart

The statistical values  $R_g$  and  $s_g$  from this series of measurements are displayed in Figure 8-11.

Collected Values		Statistics	
		$\bar{x}_g$	= 6.780000
$x_{\text{min } g}$	= 5.000	$s_g$	= 0.86699
$x_{\text{max } g}$	= 8.000	$R_g$	= 3.000
$n_{\text{tot}}$	= 25	$n_{\text{eff}}$	= 25

Figure 8-11: Result

Different formulas may be applied in order to calculate the estimate for  $u_{OBJ}$ :

as per determination method B  
(normal distribution)  $u_{OBJ} = a \cdot b = \frac{R_g}{2} \cdot 0.5 = \frac{8.0 - 5.0}{2} \cdot 0.5 = 0.75$

range method  $u_{OBJ} = a \cdot b = \frac{R_g}{d_2^*} = \frac{3.0}{3.9599} \approx 0.76$   $d_2^*$  determined  
due to MC  
simulation

where  $R_g = 2 \cdot a$

standard deviation method  $u_{OBJ} = s_g = 0.87$

### 8.2.9 Standard Uncertainty Caused by the Temperature Influence $u_T$

As is generally known, linear measures are extremely temperature-sensitive depending on the material. The actual value of a linear measurement value at standard temperature differs from the actual value at a different temperature. This deviation is caused by the thermal expansion behavior of the material. An aggravating factor is that the thermal expansion depends on the type of material. The thermal expansion coefficients of different materials are listed in tables, e.g. in VDA 5 ([70], Table A.3.2).

In practice, the following situation often occurs: The thermal expansion coefficient of the material of the gage's linear standard differs from the one of the material of the part to be inspected. If the temperature deviates from the standard temperature of 20°C, the linear expansion of the part is different from the one of the gage. This leads to the following problem: Assuming that the expansion coefficient of the part's material exceeds the one of the gage's material, the recorded measurement value for the length of the part is too high. If you took this measurement at standard temperature, the actual value of the part would be smaller. This deviation is caused by the bias due to different linear expansions. For this reason, the temperature influence affecting the measurement process must be observed. There are some particular situations where the temperature influence is negligible.

**1<sup>st</sup> case:** The measurement process operates at standard temperature and the work pieces are heated to standard temperature. There is no linear expansion caused by the temperature.

**2<sup>nd</sup> case:** The work piece and the linear standard of the gage consist of the same material and have the same temperature. There is no linear expansion caused by the temperature.

**3<sup>rd</sup> case:** The different linear expansions of the work piece and the gage are corrected by means of calculations for each measurement value (temperature compensation).

The 1<sup>st</sup> case is often not feasible because of high setup costs and operating expenses. The 2<sup>nd</sup> case may be regarded as exceptional situation. The 3<sup>rd</sup> case can hardly be

realized. The temperature influence may also be considered as an uncertainty component in the test process in order to solve this problem.

**Standard Uncertainty Caused by Temperature Influences  $u_T$  according to VDA 5**

For the maximum temperature deviation (20°C at most) occurring during the operation, the limit value  $a$  is determined for the maximum bias to be expected due to different linear expansions ([70]; formula A.3.9):

$$a = |\Delta L| + 2 \cdot u_{Rest}$$

The standard uncertainty caused by the temperature influence is calculated by multiplying the limit value  $a$  by the distribution factor  $b$  in case of a rectangular distribution ([70]; formula A.3.10):

$$u_T = a \cdot b = a \cdot \frac{1}{\sqrt{3}}$$

$\Delta L$	bias caused by different linear expansions of the work piece and gage
$u_{Rest}$	uncertainty of the expansion coefficients and temperatures

The bias caused by different linear expansions  $\Delta L$  is calculated approximately using the following formula ([70]; formula A.3.8):

$$\Delta L \approx L_{Anz;N} \cdot (\alpha_W \cdot T_W - \alpha_N \cdot T_N)$$

$L_{Anz;N}$	value displayed by the measuring device at the standard temperature of 20°C
$\alpha_W$	thermal expansion coefficient of the material of the work piece
$\alpha_N$	thermal expansion coefficient of the material of the gage
$T_W$	difference between the temperature of the work piece and the standard temperature
$T_N$	difference between the temperature of the gage and the standard temperature

The residual uncertainty  $u_{res}$  is assessed by means of the following approximate formula ([70]; formula A.3.5):

$$u_{Rest} = L_{Anz;N} \cdot \sqrt{T_N^2 \cdot u_{\alpha_N}^2 + T_W^2 \cdot u_{\alpha_W}^2 + \alpha_N^2 \cdot u_{T_N}^2 + \alpha_W^2 \cdot u_{T_W}^2}$$

$u_{\alpha_W}$	uncertainty of the thermal expansion coefficient of the work piece's material (standard value <sub>VDA</sub> : $0.1 \cdot \alpha_W$ )
$u_{\alpha_N}$	uncertainty of the thermal expansion coefficient of the gage's material (standard value <sub>VDA</sub> : $0.1 \cdot \alpha_N$ )
$u_{T_N}$	uncertainty of the gage's temperature (standard value <sub>VDA</sub> : 1 K)
$u_{T_W}$	uncertainty of the work piece's material (standard value <sub>VDA</sub> : 1 K)

Normally the tables of thermal expansion factors do not include uncertainties. Even the uncertainty of the part's and gage's temperature is hardly assessable in practice. If

these values are required, VDA 5 specifies the values that are shown in brackets above. Chapter 8.6.1.2 gives a numerical example.

### Standard Uncertainty Cause by the Temperature Influence $u_T$ as per ISO 14253 -2 [36]

VDA 5 also contains this procedure.

$\Delta T$	temperature difference
$\alpha$	expansion coefficient
$T$	mean temperature during the measurement
$u_\alpha$	standard uncertainty of the expansion coefficient of the measurement system's material
$l$	measured measure

The uncertainty of the thermal expansion coefficient of the gage's material is neglected when the measuring machine makes an automated temperature compensation but this has to be proved in each individual case.

The standard uncertainty caused by temperature influences  $u_T$  is calculated from the standard uncertainty caused by changes in the object  $u_{TD}$  and the standard uncertainty due to changes in the measurement system  $u_{TA}$ :

$$u_T = \sqrt{u_{TD}^2 + u_{TA}^2}$$

This leads to

$$u_{TD} = \Delta T \times \alpha \times l \times \frac{1}{\sqrt{3}} \quad \text{and} \quad u_{TA} = |T - 20^\circ\text{C}| \times u_\alpha \times l$$

### Standard Uncertainty Caused by the Temperature Influence $u_{TD}$ of the Differences between the Reference's Thermal Expansion and the one of the Work Piece $\Delta l$

Since the temperature influence is often hard to assess, the measurement system can be adjusted with the help of a reference part (calibration master) prior to the actual measurement. The temperature of the reference part may deviate from 20°C, the temperature it was calibrated at. This deviation must be considered when determining  $u_T$ . The measurement system is adjusted incorrectly by this value. Then the measurement object is measured. This object might have another temperature than the reference part and thus the deviation caused by the temperature difference must also be taken into account. On the basis of the difference (see Figure 8-12), it is possible to determine how the temperature depends on the standard uncertainty.

$$u_{TD} = \Delta l \times \frac{1}{\sqrt{3}}$$

**Note**

As long as the adjusted measurement system does not change its temperature considerably, it is applicable. If this temperature changes considerably, it must be readjusted.

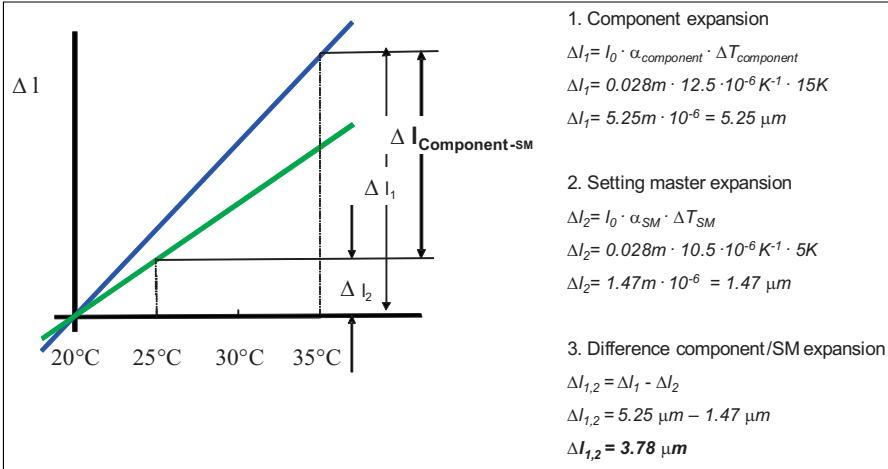


Figure 8-12: Determination of the different temperature expansions between reference part and component

**8.2.10 Standard Uncertainty Caused by Non-linearity  $u_{\text{LIN}}$**

In case of measuring devices without a linear standard, the influences caused by non-linearity must be considered. The evaluation of this influence is identical to the assessment of the bias but several points within the measuring range (several standards or calibrated reference parts) are inspected. The first standard is to lie near the lower specification limit, the second one should be in the tolerance center and the third one is to be located near the upper tolerance limit. You may apply more than three standards, however this requires a greater effort. Repeat measurements are taken from each standard in order to calculate the bias  $B_i$  ( $B_1, B_2$  and  $B_3$ ).

$B_i = |\bar{x}_{gi} - x_{mi}|$  where  $i = 1,2,3$ . The maximum value of  $B_i$  ( $B_{i\text{max}} = \max \{B_i\}$ ) is used to calculate the standard uncertainty  $u_{B_i}$ . According to determination method B, this leads to:

$$u_{B_i} = B_{i\text{max}} \times \frac{1}{\sqrt{3}}$$

**Example**

One inspector measures 3 reference parts

$x_{m1} = 30.0025 \text{ m}$

$x_{m2} = 30.005 \text{ m}$

$x_{m3} = 30.0076 \text{ m}$



Each part is measured 10 times. Table 8-6 shows the results. They are displayed in the form of a value chart (Figure 8-13) and a value plot (Figure 8-14). The numerical example is taken from VDA 5 ([70], Table A.9.3).

n	$\bar{x}_g$ Ref.	$x_{A,1}$	$x_{A,2}$	$x_{A,3}$	$x_{A,4}$	$x_{A,5}$	$x_{A,6}$	$x_{A,7}$	$x_{A,8}$	$x_{A,9}$	$x_{A,10}$	$\bar{x}_g$
1	30,002500	30,0025	30,0024	30,0024	30,0023	30,0025	30,0024	30,0023	30,0023	30,0024	30,0024	30,00239
2	30,005000	30,0050	30,0051	30,0051	30,0050	30,0052	30,0051	30,0050	30,0051	30,0051	30,0052	30,00509
3	30,007600	30,0075	30,0075	30,0077	30,0075	30,0076	30,0076	30,0076	30,0075	30,0076	30,0076	30,00757

Table 8-6: Measurement values linearity

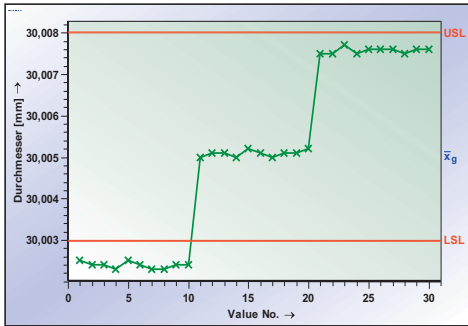


Figure 8-13: Actual value chart

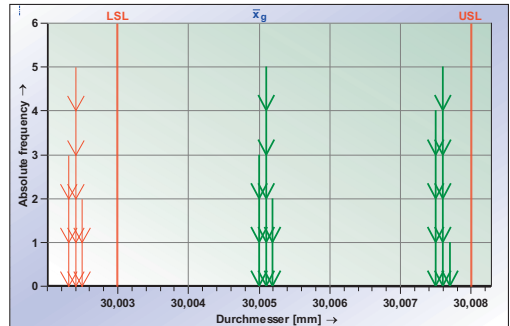


Figure 8-14: Actual value plot

The averages of the three measurements on the reference parts are displayed in Table 8-6. They lead to:

$$Bi_1 = 0.0011 \mu\text{m} \quad Bi_2 = 0.00009 \mu\text{m} \quad Bi_3 = 0.0003 \mu\text{m}$$

$Bi_1$  is the highest bias value. This value is used to calculate  $u_{LIN}$  according to determination method B.

$$u_{LIN} = \frac{Bi_{\max}}{\sqrt{3}} = \frac{0.0011}{\sqrt{3}} = 0.000635 \approx 0.6 \mu\text{m}$$

### 8.2.11 Standard Uncertainty Caused by Stability $u_{STAB}$

An analysis of the measurement process at an indefinite time does not allow for conclusions about its behavior in the future. For this reason, the measurement stability of a measurement process must be checked continuously. The intervals of these stability checks depend on the stability of the measurement process and, as described in chapter 3.5.4, must be inspected first. Then a standard or calibrated reference part is measured once or several times by means of a measuring device at the predefined intervals. Three repeat measurements have proved to be most reasonable. Statistical values such as  $R_g$  and  $s_g$  are calculated from the individual values or samples. They help to assess the standard uncertainty  $u_{Stab}$ .

Determination method B leads to (assuming a normal distribution):

$$u_{\text{STAB}} = \frac{R_g}{2} \cdot 0.50$$

If the total standard deviation is used as estimate

$$u_{\text{STAB}} = s_g$$

**Example:**

Table 8-7 contains the measurement data recorded over a longer period of time. The reference part was always measured three times at each interval and the results were displayed in a quality control chart (Figure 8-15).

i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>
1	6,002	4	6,004	7	6,003	10	6,003	13	6,002	16	6,000	19	6,001	22	6,001	25	6,000
2	6,001	5	6,004	8	6,002	11	6,001	14	6,001	17	6,001	20	6,001	23	6,002	26	6,000
3	6,001	6	6,003	9	6,002	12	6,004	15	6,002	18	5,999	21	6,000	24	6,002	27	6,001
i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>
28	6,004	31	6,002	34	6,003	37	6,002	40	6,002	43	6,004	46	6,003	49	6,002	52	6,002
29	6,004	32	6,001	35	6,001	38	6,001	41	6,000	44	6,003	47	6,002	50	6,002	53	6,002
30	6,003	33	6,002	36	6,001	39	6,002	42	6,001	45	6,003	48	6,001	51	6,000	54	6,002
i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>	i	x <sub>i</sub>
55	6,001	58	6,003	61	6,004	64	6,002	67	6,004	70	6,005	73	6,002	76		79	
56	6,002	59	6,003	62	6,003	65	6,000	68	6,003	71	6,004	74	6,001	77		80	
57	6,001	60	6,002	63	6,004	66	6,001	69	6,002	72	6,004	75	6,001	78		81	

Table 8-7: Measurement values for stability

The series of measurements leads to  
 according to determination method B  
 (based on the normal distribution)  
 range method

$$R_g = 6 \mu\text{m} \text{ and } s_g = 1.284 \mu\text{m}$$

$$u_{\text{STAB}} = \frac{6 \mu\text{m}}{2} \cdot 0.5 = 1.5 \mu\text{m}$$

$$u_{\text{STAB}} = \frac{R}{d_2^*} = \frac{6 \mu\text{m}}{1.69257} = 3.545 \mu\text{m} \approx 3.6 \mu\text{m}$$

**Note**

$d_2^*$  from Table 15.1-1,  $n = 3$  and  $r = 25$

and according to determination method A  $u_{\text{STAB}} = s_g = 2.56 \mu\text{m} \approx 2.6 \mu\text{m}$

The estimated value of the uncertainty according to the range method exceeds the estimate for the uncertainty determined by means of determination method A considerably. The reason for this is that in case of many values (here: 75 values) it is more likely that extreme minimum and maximum values occur and the range is calculated from these values.

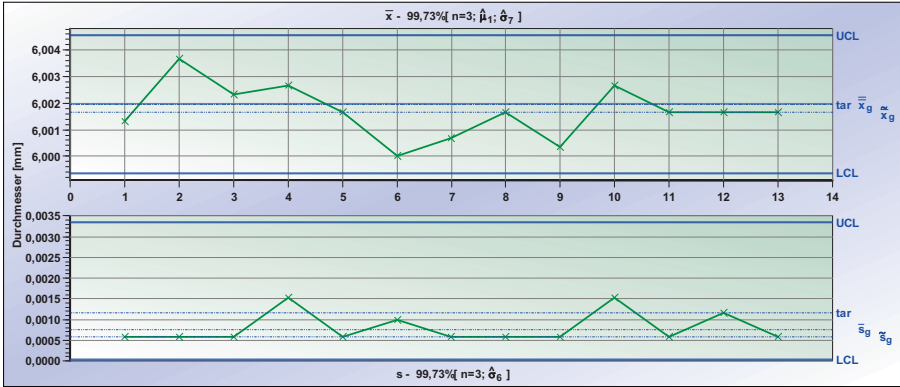


Figure 8-15: Quality control chart

**Note**

This approach includes the most influencing factors affecting the measurement process. The recorded series of measurements contains their impacts. Hence, this analysis might be used in order to evaluate the entire measurement process. In addition, only the uncertainty of the standard or the reference part must be considered. The formula of the combined standard uncertainty of the test process is:

$$u_{MP} = \sqrt{u_{CAL}^2 + u_{STAB}^2} \quad \text{where} \quad u_{CAL} = \frac{U_{CAL}}{2}$$

$U_{CAL}$  = extended measurement uncertainty of the reference part specified in the calibration certificate

### 8.3 Multiple Consideration of Uncertainty Components

Independent of the determination method used to calculate the standard uncertainty of the single components, these components must not be assessed more than once. For instance, the equipment variation  $U_{EVR}$  at the reference part may only be considered in the evaluation of the measurement system.

In the evaluation of the entire measurement process, the standard uncertainty caused by the equipment variation at the reference part  $u_{EVR}$  is compared to the one at the object. The maximum value of these two uncertainties is applied.

$$u_{EV} = \max \{u_{EVR}, u_{EVO}\}$$

The resolution must be regarded as a special case. On the one hand, the resolution must be smaller than 5% of the tolerance ( $\%RE \leq 5\% TOL$ ). This requirement must be met. On the other hand, it is possible that  $u_{RE} > u_{EVR}$ . In this case,  $u_{RE}$  must be used.