

Sample Pages

Jean-François Agassant, Pierre Avenas, Pierre J. Carreau, Bruno Vergnes,
Michel Vincent

Polymer Processing

Principles and Modeling

Book ISBN: 978-1-56990-605-7

eBook ISBN: 978-1-56990-606-4

For further information and order see

<http://www.hanser-fachbuch.de/978-1-56990-605-7>

or contact your bookseller.

Jean-François Agassant
Pierre Avenas
Pierre J. Carreau
Bruno Vergnes
Michel Vincent

Polymer Processing

Principles and Modeling

2nd Edition

Hanser Publishers, Munich

HANSER
Hanser Publications, Cincinnati

The Authors:

Jean-François Agassant,

Professor, MINES ParisTech, CEMEF, CS 10207, 06904 Sophia Antipolis Cedex, France

Pierre Avenas,

Former director of CEMEF and R&D in chemical industry, 249 rue Saint-Jacques, 75005 Paris, France

Pierre J. Carreau,

Professor Emeritus, Polytechnique Montreal, C.P. 6079 suc. Centre-Ville, Montreal, QC H3C 3A7, Canada

E-mail: pcarreau@polymtl.ca

Bruno Vergnes,

Directeur de Recherches, MINES ParisTech, CEMEF, CS 10207, 06904 Sophia Antipolis Cedex, France

Michel Vincent,

Directeur de Recherches au CNRS, MINES ParisTech, CEMEF, CS 10207, 06904 Sophia Antipolis Cedex, France

Distributed in the Americas by:

Hanser Publications

6915 Valley Avenue, Cincinnati, Ohio 45244-3029, USA

Fax: (513) 527-8801

Phone: (513) 527-8977

www.hanserpublications.com

Distributed in all other countries by:

Carl Hanser Verlag

Postfach 86 04 20, 81631 München, Germany

Fax: +49 (89) 98 48 09

www.hanser-fachbuch.de

The use of general descriptive names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone. While the advice and information in this book are believed to be true and accurate at the date of going to press, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

The final determination of the suitability of any information for the use contemplated for a given application remains the sole responsibility of the user.

Cataloging-in-Publication Data is on file with the Library of Congress

All rights reserved. No part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying or by any information storage and retrieval system, without permission in writing from the publisher.

© Carl Hanser Verlag, Munich 2017

Editor: Cheryl Hamilton

Production Management: Thomas Gerhardy

Coverconcept: Marc Müller-Bremer, www.rebranding.de, München

Coverdesign: Stephan Rönigk

Printed and bound by Hubert & Co., Göttingen

Printed in Germany

ISBN: 978-1-56990-605-7

E-Book ISBN: 978-1-56990-606-4

Contents

Foreword to the English Edition	XXVII
Preface to the Third French Edition	XXIX
Acknowledgements	XXXI
Introduction	XXXV
1 Continuum Mechanics: Review of Principles	1
1.1 Strain and Rate-of-Strain Tensor	1
1.1.1 Strain Tensor	1
1.1.1.1 Phenomenological Definitions	1
1.1.1.1.1 Extension (or Compression)	1
1.1.1.1.2 Pure Shear	2
1.1.1.2 Displacement Gradient	2
1.1.1.3 Deformation or Strain Tensor ϵ	4
1.1.1.4 Volume Variation During Deformation	5
1.1.2 Rate-of-Strain Tensor	6
1.1.3 Continuity Equation	7
1.1.3.1 Mass Balance	7
1.1.3.2 Incompressible Materials	8
1.1.4 Problems	8
1.1.4.1 Analysis of Simple Shear Flow	8
1.1.4.2 Study of Several Simple Shear Flows	9
1.1.4.2.1 Flow between Parallel Plates (Figure 1.6)	9
1.1.4.2.2 Flow in a Circular Tube (Figure 1.7)	10
1.1.4.2.3 Flow between Two Parallel Disks	10
1.1.4.2.4 Flow between a Cone and a Plate	11
1.1.4.2.5 Couette Flow	12
1.1.4.3 Pure Elongational Flow	12
1.1.4.3.1 Simple Elongation	12

1.1.4.3.2	Biaxial Stretching: Bubble Inflation	13
1.2	Stresses and Force Balances	14
1.2.1	Stress Tensor	14
1.2.1.1	Phenomenological Definitions	14
1.2.1.1.1	Extension (or Compression) (Figure 1.13)	14
1.2.1.1.2	Simple Shear (Figure 1.14)	15
1.2.1.2	Stress Vector	15
1.2.1.3	Stress Tensor	16
1.2.1.4	Isotropic Stress or Hydrostatic Pressure	17
1.2.1.5	Deviatoric Stress Tensor	17
1.2.2	Equation of Motion	18
1.2.2.1	Force Balances	18
1.2.2.2	Torque Balances	20
1.2.3	Problems	21
1.2.3.1	Shear Stress at the Surface of a Tube	21
1.2.3.2	Stresses in a Shell	21
1.3	General Equations of Mechanics	22
1.3.1	General Case	22
1.3.2	Incompressibility	23
1.3.3	Planar Flow	23
1.3.4	Problem: Stress Tensor in Simple Shear Flow	24
1.4	Appendices	25
1.4.1	Appendix 1: Basic Formulae	25
1.4.1.1	Cylindrical Coordinates	25
1.4.1.2	Spherical Coordinates	27
1.4.2	Appendix 2: Invariants of a Tensor	28
1.4.2.1	Definitions	28
1.4.2.2	Invariants Used in Fluid Mechanics	29
References		31
2	Rheological Behavior of Molten Polymers	33
2.1	Viscosity: Equations for Newtonian Fluids	33
2.1.1	Basic Experiment of Newtonian Behavior	33
2.1.1.1	Phenomenological Definition of Newton (1713)	33
2.1.1.2	Experiment of Trouton: Concept of Elongational Viscosity	34
2.1.2	Generalization to Three Dimensions	35
2.1.2.1	Constitutive Equation	35
2.1.2.2	Simple Shear Flow	35
2.1.2.3	Uniaxial Extensional Flow	36
2.1.3	Magnitudes of the Forces Involved	36

2.1.3.1	Units of Viscosity and Orders of Magnitude	36
2.1.3.2	Reynolds Number	37
2.1.3.3	Effect of Gravity	38
2.1.4	Navier-Stokes Equations	38
2.1.5	Problems	39
2.1.5.1	Simple Shear Flow	39
2.1.5.2	Planar Pressure Flow	40
2.1.5.3	Superposition of a Simple Shear Flow and a Planar Pressure Flow	41
2.1.5.4	Pressure Flow in a Tube	43
2.1.5.5	Simple Shear between Two Parallel Disks	44
2.1.5.6	Couette Flow	44
2.1.5.7	Flow in a Dihedron	46
2.1.5.8	Flow in a Cone	47
2.2	Shear-Thinning Behavior	48
2.2.1	Phenomenological Description	48
2.2.2	Rheological Models in One Dimension	48
2.2.2.1	Power-Law Model	49
2.2.2.2	Cross Model	50
2.2.2.3	Carreau Model	50
2.2.3	Physical Explanation of the Shear-Thinning Behavior of Polymers	50
2.2.4	Three-Dimensional Constitutive Equations	52
2.2.5	Applications of the Power Law to Simple Flows	53
2.2.5.1	Simple Shear Flow	53
2.2.5.2	Pressure Flow in a Tube	53
2.2.6	Problems in Power-Law Fluids	55
2.2.6.1	Simple Shear Flow between Parallel Plates	55
2.2.6.2	Pressure Flow in a Tube	56
2.2.6.3	Planar Pressure Flow	57
2.2.6.4	Superposition of a Simple Shear Flow and a Planar Pressure Flow	58
2.2.6.5	Simple Shear Flow between Disks	59
2.2.6.6	Couette Flow	60
2.3	Behavior of Filled Polymers	60
2.3.1	Rheological Behavior of Suspensions	61
2.3.1.1	Dilute Suspensions of Spheres	61
2.3.1.2	Concentrated Suspensions of Spheres	62
2.3.1.3	Special Case of Fibers	63
2.3.1.3.1	Orientation	63
2.3.1.3.2	Rheological Behavior	67
2.3.2	Yield Stress Fluids	68

2.3.3	Problem: Pressure Flow of a Yield Stress Fluid in a Pipe	71
2.4	Viscoelastic Behavior	72
2.4.1	Physical Phenomena.	72
2.4.1.1	Extrudate Swell	72
2.4.1.2	Weissenberg Effect	73
2.4.1.3	Time-Dependent Behavior	73
2.4.1.3.1	Stress Retardation and Relaxation	74
2.4.1.3.2	Recovery of Deformation after Cessation of Stress	74
2.4.1.3.3	Response of a Polymer to a Sinusoidal Motion	75
2.4.2	Linear Viscoelasticity and the Maxwell Model	75
2.4.2.1	General Information on Linear Viscoelastic Models	75
2.4.2.2	Behavior of a Maxwell Element	77
2.4.2.3	Qualitative Interpretation of Time-Dependent Phenomena	78
2.4.2.3.1	Stress Relaxation (Figure 2.30)	78
2.4.2.3.2	Stress Retardation (Figure 2.31)	78
2.4.2.3.3	Strain Recovery	78
2.4.2.3.4	Response to a Periodic Strain	79
2.4.3	Normal Stress Difference in Simple Shear	81
2.4.4	Extrudate Swell	83
2.4.5	Convected Maxwell Model	85
2.4.5.1	Transient Behavior	86
2.4.5.2	Viscometric Functions	86
2.4.5.3	Elongational Viscosity	87
2.4.6	Viscoelastic Dimensionless Numbers	88
2.4.7	Physical Interpretation of the Viscoelastic Behavior of Polymer Melts	88
2.4.7.1	Rouse Model (1953)	89
2.4.7.2	Temporary Network Models	90
2.4.7.3	Models of Cooperative Motion of a Chain and Its Neighbors	90
2.4.7.4	Reptation Models	90
2.4.7.5	Pom-Pom Models	91
2.4.8	Some Viscoelastic Constitutive Equations	92
2.4.8.1	Different Types of Viscoelastic Constitutive Equations	92
2.4.8.1.1	Equations with Memory Function or Integral Constitutive Equations	92
2.4.8.1.2	Differential Constitutive Equations	93
2.4.8.2	Choice of a Rheological Model	95
2.4.9	Problems in the Convected Maxwell Model	95
2.4.9.1	Maxwell Fluid in Simple Shear	95

2.4.9.2	Shear Flow of a Maxwell Fluid between Parallel Disks. . .	97
2.4.9.3	Couette Flow of a Maxwell Fluid	101
2.4.9.4	Stretching of a Maxwell Fluid	103
2.5	Measurement of the Rheological Behavior of Polymer Melts	108
2.5.1	Capillary Rheometer: Viscosity Measurements.	108
2.5.1.1	Principle of the Measurements	108
2.5.1.2	Obtaining a Viscosity Curve	110
2.5.1.3	Influence of Temperature.	114
2.5.1.3.1	Arrhenius Equation	115
2.5.1.3.2	WLF Equation	116
2.5.1.3.3	Master Curves	117
2.5.1.4	Influence of Pressure	118
2.5.2	Slit Die Rheometer	119
2.5.3	Flow with a Wall Slip	121
2.5.4	Cone-and-Plate Rheometer.	123
2.5.4.1	Presentation of the Cone-and-Plate Rheometer	123
2.5.4.2	Steady Shear	123
2.5.4.3	Oscillatory Shear (SAOS)	126
2.5.4.4	Transient Modes	129
2.5.5	Parallel-Plate Rheometer	130
2.5.5.1	Steady Shear	130
2.5.5.2	Oscillatory Shear (SAOS)	130
2.5.6	Elongational Rheometry.	131
2.5.6.1	Difficulties in Elongational Viscosity Measurements	131
2.5.6.2	Elongational Rheometers	132
2.5.6.3	Other Measurement Methods	133
2.5.6.3.1	Isothermal Stretching.	134
2.5.6.3.2	Converging Flows	134
2.5.7	Notions of Rheo-optics	135
2.5.7.1	Flow Birefringence	136
2.5.7.1.1	Measurement Principle and Experimental Setup	136
2.5.7.1.2	Example of Experimental Results	138
2.5.7.2	Laser Doppler Velocimetry.	140
2.5.7.2.1	Measurement Principle and Experimental Setup	140
2.5.7.2.2	Example of Results	141
2.5.8	Perspective.	142
2.6	Appendices.	142
2.6.1	Appendix 1: Physics of Viscosity	142
2.6.1.1	Eyring Theory	142
2.6.1.2	Molecular Weight Dependence of the Viscosity of Polymers	144

2.6.1.2.1	Viscosity of Polymers Having a Molecular Weight Less than M_c	145
2.6.1.2.2	Viscosity of Polymers Having a Molecular Weight Higher than M_c	147
2.6.1.3	Free Volume Theory	148
2.6.2	Appendix 2: An Approach to Viscoelasticity: Elastic Dumbbell Model	149
2.6.2.1	Interest of the Dumbbell Models	149
2.6.2.2	Model Description	150
2.6.2.3	Dumbbell in Simple Shear	151
2.6.2.3.1	Hydrodynamic Actions	151
2.6.2.3.2	Force due to Brownian Motion	152
2.6.2.3.3	Balance of Forces and Conservation of the Number of Molecules	152
2.6.2.3.4	Average Deformation of the Macromolecule	153
2.6.2.3.5	Comments	154
2.6.2.4	Macromolecule Deformation in Complex Flows	154
2.6.2.5	Macromolecule Deformation in Planar Extension	156
2.6.2.6	Concluding Remarks	157
2.6.3	Appendix 3: Material and Convected Derivatives	158
2.6.3.1	Substantial or Material Derivative of a Tensor	158
2.6.3.2	Convected Derivative of a Tensor	158
2.6.3.3	Special Case of the Rotation of a Disk about Its Axis	160
2.6.4	Appendix 4: Rabinowitsch Correction (Rabinowitsch, 1929)	162
2.6.5	Appendix 5: Flow of a Viscoelastic Fluid in a Cone-and-Plate Geometry	163
2.6.5.1	Kinematics Hypotheses	164
2.6.5.2	Viscometric Functions	165
2.6.5.3	Dynamic Equilibrium of the System	165
2.6.5.4	Small Cone Angle Limit	166
2.6.6	Appendix 6: Viscometric Flows	168
	References	169
3	Energy and Heat Transfer in Polymer Processes	177
3.1	Basic Notions on Heat Transfer	177
3.1.1	First Law of Thermodynamics	177
3.1.2	Heat Received by the System	178
3.1.3	Power Generated by Internal Forces	178
3.1.3.1	Work Done by Deformation	178
3.1.3.1.1	Extension or compression	178
3.1.3.1.2	Simple Shear	179

3.1.3.2	Generalization	179
3.1.3.3	Power Generated by Internal Forces (Dissipated Power)	179
3.1.3.4	Newtonian and Shear-Thinning, Power-Law Liquids	180
3.1.4	Equation of Energy	180
3.1.5	Internal Energy	181
3.1.5.1	Temperature-Dependent Internal Energy, e	181
3.1.5.2	Compressible Materials	184
3.1.5.3	Change of State or Chemical Reaction	184
3.1.6	Boundary Conditions	185
3.1.6.1	Mathematical Conditions	185
3.1.6.2	Conditions Depending on the Environment	185
3.1.6.2.1	Polymer in Contact with a Metallic Surface	185
3.1.6.2.2	Polymer in Contact with a Fluid (Air or Water)	188
3.1.7	Solutions of the Heat Transfer Equation	189
3.2	Cooling in Molds, in Air, and in Water	190
3.2.1	Context	190
3.2.2	Heat Transfer Equation	190
3.2.2.1	Body at Rest	190
3.2.2.2	Body in Motion	191
3.2.3	Heat Penetration Thickness	191
3.2.4	Interfacial Temperature	193
3.2.4.1	Conductive Heat Transfer: Notion of Effusivity	193
3.2.4.2	Conductive and Convective Heat Transfer	196
3.2.5	Heating (or Cooling) of a Plate	198
3.2.5.1	Isothermal Boundary Conditions	198
3.2.5.1.1	Exact Solution	198
3.2.5.1.2	Approximate Solution	199
3.2.5.2	Convective Boundary Conditions	200
3.2.5.2.1	Exact Solution	200
3.2.5.2.2	Approximate Solution	202
3.3	Polymer Flow and Heat Transfer	203
3.3.1	Importance of Viscous Heating: The Brinkman Number	204
3.3.2	Notion of a Thermal Regime	204
3.3.3	The Equations	205
3.3.3.1	Energy Equation	205
3.3.3.2	Calculation of the Dissipated Heat, \dot{W}	206
3.3.3.2.1	Newtonian Behavior	206
3.3.3.2.2	Shear-Thinning, Power-Law Behavior	206
3.3.4	Equilibrium Regime	207
3.3.4.1	Equilibrium Regime for a Newtonian Polymer	207
3.3.4.1.1	Constant Temperature at the Walls, $T(R) = T_w$	207

3.3.4.1.2	Convective Boundary Condition	208
3.3.4.2	Equilibrium Regime for a Power-Law Polymer and a Constant Wall Temperature	210
3.3.5	Adiabatic Regime	211
3.3.6	Transition Regime for a Newtonian Fluid.	213
3.3.6.1	Average Temperature with a Convective Boundary Condition	213
3.3.6.2	Evaluation of the Nusselt Number (or of the Heat Transfer Coefficient).	214
3.3.6.2.1	Expression for Nu_{eq}	214
3.3.6.2.2	Control Temperature Equal to the Initial Polymer Temperature	215
3.3.6.2.3	Control Temperature Different from the Initial Polymer Temperature	216
3.3.7	Transition Regime with a Power-Law Fluid	219
3.3.8	Comparison with an Exact Solution	220
3.3.8.1	Calculations without Mechanical–Thermal Coupling.	220
3.3.8.1.1	Newtonian Polymer.	220
3.3.8.1.2	Shear-Thinning Polymer.	223
3.3.8.2	Computations with Thermal Coupling	224
3.3.9	Other Flow Geometries.	224
3.3.9.1	Simple Shear Flow between Parallel Plates	224
3.3.9.1.1	Thermal Equilibrium Regime	225
3.3.9.1.2	Adiabatic Regime	226
3.3.9.1.3	Transition Regime.	226
3.3.9.2	Heat Generation in Planar Pressure Flow	227
3.3.10	Application to Flat Die Extrusion	228
3.3.11	Conclusion.	231
3.4	Appendices.	231
3.4.1	Appendix 1: Convective Heat Transfer	231
3.4.1.1	Free and Forced Convection.	231
3.4.1.2	The Bénard Problem	232
3.4.1.2.1	Description of the Experiments.	232
3.4.1.2.2	Determination of ΔT_c (Rayleigh, 1916).	233
3.4.1.3	Heat Transfer by Free Convection.	235
3.4.1.3.1	General Principles	235
3.4.1.3.2	Horizontal Cylinder	235
3.4.1.3.3	Vertical Plate or Cylinder.	236
3.4.1.3.4	Horizontal Plate.	236
3.4.1.4	Example: Determination of the Heat Transfer Coefficient in Free Convection	237

3.4.1.4.1	Introduction	237
3.4.1.4.2	Physical Properties of Air and Water	238
3.4.1.4.3	Example	239
3.4.1.5	Forced Convection	239
3.4.1.5.1	Introduction	239
3.4.1.5.2	General Relationships	240
3.4.1.5.3	Sphere	240
3.4.1.5.4	Cylinder Perpendicular to the Flow Stream	240
3.4.1.5.5	Plate or Cylinder Parallel to the Flow Stream	240
3.4.1.5.6	Example: Determination of the Heat Transfer Coefficient in Forced Convection	241
3.4.2	Appendix 2: Radiation Heat Transfer	242
3.4.2.1	Blackbody	242
3.4.2.2	Nonblackbodies	243
3.4.2.2.1	Absorption of Nonblackbodies	243
3.4.2.2.2	Radiation Emitted by a Nonblackbody	243
3.4.2.3	Radiation Heat Exchange between Gray Bodies	244
3.4.2.3.1	Generalities	244
3.4.2.3.2	Examples	245
3.4.2.4	Determination of Radiation Heat Transfer Coefficient	246
3.4.3	Appendix 3: Internal Energy for Compressible Materials	246
	References	248
4	Approximations and Calculation Methods	251
4.1	Equations for Polymer Processing	251
4.2	Choice of a Relevant Rheological Constitutive Equation	253
4.3	Choice of Boundary Conditions	255
4.3.1	Kinematics Boundary Conditions	255
4.3.2	Heat Transfer Boundary Conditions	256
4.3.3	Inlet Conditions	256
4.3.4	Exit Conditions	256
4.4	Approximation Methods	257
4.4.1	Approximations Concerning the Geometry of the Flow	257
4.4.1.1	Unwinding or Flattening of an Annular or a Helical Geometry	257
4.4.1.2	Decomposition of Complex Flow Geometry in Several Simple Flows	258
4.4.2	Kinematics Approximations	259
4.4.2.1	Lubrication Approximations	259
4.4.2.2	Hele-Shaw Approximations	260
4.4.2.3	Approximation of a Slender Body (or Thin Film)	263

4.4.2.4	Important Remark.....	265
4.4.3	Approximations for the Temperature.....	265
4.4.4	Conclusion and Application Example.....	266
4.4.5	Problems.....	268
4.4.5.1	Flow in a Dihedron.....	268
4.4.5.2	Flow in a Cone.....	271
4.5	Pressure Buildup in Polymer Flows: Hydrodynamics Bearings.....	272
4.5.1	Introduction.....	272
4.5.2	Qualitative Analysis of Some Hydrodynamics Bearings.....	273
4.5.2.1	Rayleigh Bearing.....	273
4.5.2.2	Reynolds Bearing.....	273
4.5.2.3	Flow between Two Rolls.....	274
4.5.3	Pressure Generated by a Sudden Flow Restriction (Rayleigh Bearing).....	275
4.5.4	Flow Calculation in a Bearing of Variable Gap: the Reynolds Equation.....	276
4.5.5	Problem: Reynolds Bearing.....	277
4.6	Calculation Methods.....	279
4.6.1	Calculation Methods as Functions of the Type of Flow.....	279
4.6.1.1	Simple Shear or Simple Stretching Isothermal Flows.....	279
4.6.1.2	Unidirectional Isothermal Flows.....	279
4.6.1.3	Nonisothermal Shear or Elongation Unidirectional Flows.....	280
4.6.1.4	Bidirectional Thin Layer Flows (Isothermal or Nonisothermal).....	280
4.6.1.5	2D or 3D Flows.....	280
4.6.2	Solution of Unidirectional Flows: Slab Method (or Incremental Method).....	281
4.6.3	Solution of the Hele-Shaw Equations.....	282
4.6.3.1	Newtonian Isothermal Case.....	282
4.6.3.2	Non-Newtonian Isothermal Viscous Case.....	284
4.6.3.3	Nonisothermal Case (Average Temperature Solution) ..	285
4.6.4	2D and 3D Viscous Flow Calculations with a Finite Elements Method.....	287
4.6.4.1	Mechanical Equations.....	287
4.6.4.2	Meshing.....	287
4.6.4.3	Finite Elements Solution.....	289
4.6.4.4	Finite Elements Solution of the Energy Equation.....	290
4.6.5	Isothermal Flow Viscoelastic Computations.....	292
4.6.5.1	Direct Solution Methods.....	292
4.6.5.2	Iterative Methods.....	293

4.7	Appendix	295
4.7.1	Appendix 1: Analysis of the Lubrication Approximations	295
4.7.1.1	Introduction	295
4.7.1.2	Analysis of the Relative Weight of the Terms of the Rate-of-Strain Tensor	295
4.7.1.3	Simplification of the Equations of Motion	296
4.7.1.4	Validity of the Lubrication Approximations	297
	References	298
5	Single-Screw Extrusion and Die Flows	301
5.1	Single-Screw Extrusion	303
5.1.1	Geometric and Kinematic Description	303
5.1.1.1	The Different Zones of the Extruder	303
5.1.1.2	Geometry of the Screw	304
5.1.1.3	Description of the Screw Channel	305
5.1.1.4	Classical Approximations	306
5.1.1.4.1	Approximation of a Fixed Barrel and a Rotating Screw	306
5.1.1.4.2	Unwound Screw Channel	307
5.1.1.4.3	Relative Velocity of the Barrel	308
5.1.1.5	Reference Extruder	309
5.1.2	Feeding Zone	309
5.1.2.1	Solid Polymer Conveying	309
5.1.2.2	Polymer–Metal Friction	310
5.1.2.3	Archimedes' Screw	311
5.1.2.4	Model of Darnell and Moll (1956)	313
5.1.2.5	Flow Rate Calculation and Optimization	315
5.1.2.6	Role of Pressure	317
5.1.2.7	Technological Consequences	319
5.1.2.8	Model Improvements	320
5.1.3	Melting Zone	322
5.1.3.1	Physical Description of the Phenomena	322
5.1.3.1.1	Experimental Observations	322
5.1.3.1.2	Delay Zone (Kacir and Tadmor, 1972)	324
5.1.3.1.3	Initiation of the Melting by Melt Pool	325
5.1.3.1.4	Melting Mechanism by Melt Pool	325
5.1.3.2	Initiation of the Melting Process by Melt Pool	326
5.1.3.3	Melting Model of Tadmor and Klein (1970)	328
5.1.3.3.1	Melting Rate	328
5.1.3.3.2	Changes Induced by the Clearance between the Screw and the Barrel	331
5.1.3.3.3	Length of the Melting Zone; Role of Compression	332

5.1.3.4	Other Models	335
5.1.3.5	Technological Consequences: Barrier Screws	336
5.1.4	Flow of the Molten Polymer	339
5.1.4.1	Pumping Zone	339
5.1.4.1.1	Review of the Geometry	340
5.1.4.1.2	Flow Equations	340
5.1.4.1.3	Study of the Transverse Flow	341
5.1.4.1.4	Study of the Longitudinal Flow	343
5.1.4.1.5	Concept of Residence Time Distribution	345
5.1.4.2	Compression Zone	348
5.1.4.3	Role of the Screw/Barrel Clearance	350
5.1.4.4	Study of Thermal Phenomena	352
5.1.4.5	Concept of Characteristic curves	355
5.1.4.6	Model Improvements	357
5.1.4.7	Technological Consequences	358
5.1.4.7.1	Degassing Extruders (Two-Stage Vented Screws)	358
5.1.4.7.2	Mixing Elements	359
5.1.5	Overall Model of Single-Screw Extrusion	361
5.1.5.1	Introduction	361
5.1.5.2	Examples of Results	361
5.1.5.2.1	Reference Extruder	361
5.1.5.2.2	Optimization of the Pumping Zone	364
5.1.5.3	Conclusions	365
5.1.6	Extrusion Problems	366
5.1.6.1	Initiation of the Melting by Melt Pool	366
5.1.6.2	Melting Regime by Melt Pool	368
5.1.6.3	Criteria for Choosing an Extruder	373
5.2	Extrusion Dies	378
5.2.1	Introduction: Role of an Extrusion Die	378
5.2.2	Description of the Encountered Geometries	378
5.2.2.1	Film-Blowing Dies	378
5.2.2.2	Pipe Dies	379
5.2.2.3	Plate Dies (or Flat Dies)	380
5.2.2.4	Profile Dies	381
5.2.2.5	Wire-Coating Dies	381
5.2.3	Assumptions and Calculation Methods Revisited	382
5.2.4	Examples of Results	383
5.2.4.1	Film-Blowing Dies	383
5.2.4.2	Pipe Dies	388
5.2.4.3	Flat Dies	392
5.2.4.4	Wire-Coating Dies	395

5.2.4.5	Profile Dies	399
5.2.5	Conclusion	402
5.2.6	Die Problems	403
5.2.6.1	Flow in a Flat T-die	403
5.2.6.2	Flow in a Flat Coat-Hanger Die	405
5.3	Multilayer Flows	408
5.3.1	Interest of Multilayer Flows and Related Problems	408
5.3.2	Study of the Steady Flow of Two Viscous Fluids between Parallel Plates	410
5.3.2.1	Continuity Conditions at the Interface	411
5.3.2.2	Isothermal Newtonian Two-Layer Flow	411
5.3.2.3	Generalization to Power-Law Behavior	414
5.3.3	Flat Die Coextrusion	416
5.3.3.1	Process Description	416
5.3.3.2	One-Dimensional Approach	417
5.3.3.3	Two-Dimensional Approach	420
5.3.3.4	Two-Dimensional Hele-Shaw Approach	422
5.3.4	Coextrusion Die Problems	423
5.3.4.1	Three-Layer Coextrusion Flow between Parallel Plates	423
5.3.4.2	Coextrusion Flow in a Capillary	425
5.4	Appendix	426
5.4.1	Appendix 1: Calculation of Solid Velocity in Single-Screw Extrusion	426
References.	427
6	Twin-Screw Extrusion and Applications	433
6.1	General Description of Twin-Screw Extrusion Process	433
6.1.1	Different Types of Twin-Screw Extruders	433
6.1.2	Flow Types	434
6.1.3	Specific Features of Corotating Twin-Screw Extrusion	436
6.1.4	Geometry of Screws and Barrel	438
6.1.5	Classical Approximations	442
6.1.6	Different Modeling Approaches	443
6.1.7	Reference Extruder	443
6.2	Solid Conveying and Melting	445
6.2.1	Solid Conveying Zone	445
6.2.2	Melting Zone	447
6.3	Melt Flow	451
6.3.1	Right- and Left-Handed Screw Elements	452
6.3.1.1	One-Dimensional Models	452
6.3.1.2	Two-Dimensional Models	456

6.3.1.3	Three-Dimensional Models	457
6.3.1.4	Thermal Effects	459
6.3.2	Mixing Elements	460
6.3.2.1	One-Dimensional Models	461
6.3.2.2	Two-Dimensional Models	464
6.3.2.3	Three-Dimensional Models	467
6.4	Global Model of Twin-Screw Extrusion	469
6.4.1	General Description	469
6.4.2	Residence Time Distribution	473
6.4.3	Examples of Results	477
6.5	Application to the Production of Polymer Blends	481
6.5.1	Basic Mechanisms	481
6.5.1.1	Mechanisms of Rupture	482
6.5.1.2	Mechanisms of Coalescence	484
6.5.2	Modeling along the Extruder and Examples of Results	485
6.6	Application to Compounding Operations	488
6.6.1	Different Types of Mixing	488
6.6.2	Distributive Mixing	489
6.6.3	Dispersive Mixing: Application to the Production of Nanocomposites	492
6.7	Application to Reactive Extrusion	499
6.8	Optimization and Scale-Up	506
6.9	Conclusion	508
6.10	Problem: Simplified Model of the Flow around a Kneading Disk	508
	References	512
7	Injection Molding	521
7.1	Description	521
7.2	Filling Stage	526
7.2.1	Peculiarities of the Filling Phase	526
7.2.2	Main Hypotheses and Governing Equations	526
7.2.2.1	Purely Viscous Flow Behavior	526
7.2.2.2	Incompressibility	527
7.2.2.3	Negligible Gravitational and Inertial Forces	527
7.2.2.4	Equations	527
7.2.3	Unidirectional Flows	528
7.2.3.1	Introduction	528
7.2.3.2	Filling of a Center-Gated Disk Mold	529
7.2.3.2.1	Newtonian Isothermal Behavior	530
7.2.3.2.2	Isothermal Shear-Thinning Behavior	531
7.2.3.2.3	Nonisothermal Generalized Newtonian Behavior	532

7.2.4	Thin Flow or Hele-Shaw Models	540
7.2.5	3D Computations.	544
7.3	Packing and Holding Phase	548
7.3.1	Introduction.	548
7.3.2	Simplified Calculations of the Packing Phase	549
7.3.3	Physical Data for the Packing-Holding Calculations	551
7.3.3.1	Measurements of PVT Data	551
7.3.3.2	Modeling	552
7.3.4	Calculations.	553
7.3.4.1	Thin-Flow Approaches	553
7.3.4.2	3D Computations.	556
7.3.5	Conclusions.	557
7.4	Residual Stresses and Deformations	558
7.4.1	Introduction.	558
7.4.2	Main Physical Phenomena Involved	558
7.4.2.1	Thermal Shrinkage	558
7.4.2.2	Frozen-In Stresses.	561
7.4.3	Measurement of Residual Stresses	562
7.4.4	Calculations of Residual Stresses	563
7.5	Nonstandard Injection-Molding Techniques.	564
7.5.1	Gas-Assisted Injection Molding (GAIM)	564
7.5.2	Water-Assisted Injection Molding (WAIM).	566
7.5.3	Multicomponent Injection Molding.	567
7.6	Injection of Short Fiber Reinforced Polymers.	569
7.7	Conclusion	571
7.8	Problems.	572
7.8.1	Filling of a Center-Gated Disk	572
7.8.2	Balancing of a Multicavity Mold	575
	References.	580
8	Calendering	587
8.1	Introduction.	587
8.2	Rigid Film Calendering Process.	588
8.2.1	Presentation.	588
8.2.2	Calendering Problems	589
8.2.3	Aim of Calendering Process Modeling	591
8.2.4	Kinematics of Calendering.	591
8.2.5	Isothermal Newtonian Model Based on Lubrication Approximations.	594
8.2.5.1	Reynolds Equation	594
8.2.5.2	Spread Height Calculation	594

8.2.5.3	Roll Separating Force and Torque Exerted on the Roll	596
8.2.6	More General Newtonian Models	597
8.2.6.1	Two-Dimensional Model	597
8.2.6.2	Influence of Slippage between the Polymer and the Rolls	599
8.2.6.3	Calendering Analysis When Introducing a Velocity Differential between the Rolls	600
8.2.6.4	Conclusions of the Different Newtonian Models	601
8.2.7	Shear-Thinning Calendering Model	601
8.2.7.1	Generalized Reynolds Equation	602
8.2.7.2	Integrated Generalized Reynolds Equation	603
8.2.8	Thermal Effects in Calendering	604
8.2.9	Viscoelastic Models	608
8.2.10	Use of Calendering Models	608
8.3	Postextrusion Calendering Process	610
8.3.1	Presentation	610
8.3.2	Process Modeling	611
8.3.2.1	Pressure Field Calculations	611
8.3.2.2	Temperature Field Calculations	612
8.4	Appendix	614
8.4.1	Appendix 1: Calculations of Two-Dimensional Flow in the Calender Bank by a Finite Element Method	614
8.4.1.1	The Stokes Equations in Terms of the Stream and Vorticity Functions	614
8.4.1.2	Solving the Stream and Vorticity Equations for the 2D Calendering Problem (Agassant and Espy, 1985)	615
	References	616
9	Polymer Stretching Processes	619
9.1	Introduction	619
9.2	Fiber Spinning	619
9.2.1	Different Fiber Spinning Situations	619
9.2.2	Isothermal Melt Spinning of a Newtonian Fluid	621
9.2.2.1	Kinematics Hypotheses	622
9.2.2.2	Set of Equations	623
9.2.2.3	Solution for Isothermal Newtonian Fiber Spinning	623
9.2.2.4	Application Examples	624
9.2.2.5	Validity of the Approximations Used	625
9.2.2.5.1	Neglecting the Shear Component	625
9.2.2.5.2	Neglecting the Gravitational (Mass) Force	625
9.2.2.5.3	Neglecting the Inertia Force	626

9.2.3	Isothermal Melt Spinning of a Viscoelastic Fluid	627
9.2.3.1	Equations	627
9.2.3.2	Dimensionless Equations	628
9.2.3.3	Solution	629
9.2.3.4	Results	630
9.2.4	Drawing of a Viscous Fluid in Nonisothermal Conditions	632
9.2.4.1	Mechanical Equations	632
9.2.4.1.1	Equations of Motion	632
9.2.4.1.2	Force Balance at the Filament Surface	633
9.2.4.1.3	Newtonian Hypothesis	634
9.2.4.2	Heat Transfer Equation	634
9.2.4.2.1	Forced Convection Term	635
9.2.4.2.2	Radiative Heat Transfer Coefficient	636
9.2.4.2.3	Viscous Dissipation Rate during Drawing	636
9.2.4.3	Solution for the Momentum and Heat Transfer Equations	637
9.2.4.4	Results	637
9.2.5	More General Models of Fiber Spinning	639
9.3	Biaxial Drawing	640
9.3.1	Introduction	640
9.3.2	Biaxial Stretching of a Newtonian Liquid	640
9.4	Cast-Film Process	642
9.4.1	Presentation	642
9.4.2	Different Kinematics Approaches	643
9.4.2.1	Two-Dimensional Membrane Approach	643
9.4.2.2	One-Dimensional Membrane Approach	644
9.4.2.3	One-Dimensional Approach	645
9.4.3	One-Dimensional Newtonian Model	645
9.4.4	One-Dimensional Membrane Model	646
9.4.4.1	Equations of the Newtonian Model	646
9.4.4.1.1	Stress Tensor	646
9.4.4.1.2	Equations	647
9.4.4.1.3	Boundary Conditions	647
9.4.4.2	Results of the One-Dimensional Newtonian Membrane Model	648
9.4.4.3	Equations of a Viscoelastic Model	649
9.4.4.4	Results of the One-Dimensional Viscoelastic Membrane Model	651
9.4.5	Two-Dimensional Membrane Model	652
9.4.5.1	Equations of the Problem	652
9.4.5.2	Results of the Two-Dimensional Membrane Model	653

	9.4.5.3	Nonisothermal Model	655
	9.4.6	Conclusions	657
	9.4.7	Problems	658
	9.4.7.1	Drawing of a Constant-Width Film	658
	9.4.7.2	Extrusion of Tubes	660
9.5		Film-Blowing Process	661
	9.5.1	Process Description	661
	9.5.2	Film Geometry	664
	9.5.3	Equations of the Film-Blowing Process	665
	9.5.3.1	Kinematics of Bubble Formation	665
	9.5.3.2	Stresses Acting on the Bubble	665
	9.5.3.2.1	Force Balance in the Drawing Direction and Meridian Stress	665
	9.5.3.2.2	Force Balance Perpendicular to the Film	667
	9.5.3.2.3	Order of Magnitude of the Stress Components	668
	9.5.3.3	Heat Balance Equations	669
	9.5.4	Nonisothermal Newtonian Model	670
	9.5.4.1	Equations	670
	9.5.4.2	Examples of Results	671
	9.5.5	Nonisothermal Viscoelastic Model	673
	9.5.5.1	Equations	673
	9.5.5.2	Examples of Results	674
	9.5.6	A Semiempirical Model of the Blown-Film Process	677
	9.5.7	Conclusions	678
9.6		Manufacture of Hollow Plastic Bodies	679
	9.6.1	Various Blow-Molding Processes	679
	9.6.1.1	Extrusion Blow Molding	679
	9.6.1.2	Stretch Blow-Molding Process	680
	9.6.1.3	Problems Encountered in Blow Molding	680
	9.6.2	Modeling of Extrusion Blow Molding	681
	9.6.2.1	Membrane or Thick Shell?	681
	9.6.2.2	Choice of Rheological Behavior	683
	9.6.2.3	Application to the Blowing of a Complex Hollow Part	685
	9.6.2.3.1	Curvilinear Coordinates	686
	9.6.2.3.2	Dynamic Equilibrium of the Membrane	687
	9.6.2.3.3	Boundary Conditions for the Pressure	688
	9.6.2.3.4	Example	690
	9.6.3	Stretch Blow-Molding Process	692
	9.6.3.1	Introduction	692
	9.6.3.2	Process Modeling	692
	9.6.3.2.1	Model	693

9.6.3.2.2	Boundary Conditions	694
9.6.3.2.3	Numerical Solution	694
9.6.3.3	Example	695
9.6.4	Conclusions	696
9.6.5	Problems	697
9.6.5.1	Inflation of a Newtonian Spherical Membrane	697
9.6.5.2	Blowing of a Tubular Newtonian Membrane of Constant Length	698
9.6.5.3	Blowing of a Thick Newtonian Tube of Constant Length	701
9.7	Appendices	704
9.7.1	Appendix 1: Solution of the Isothermal Cast-Film Equations	704
9.7.1.1	One-Dimensional Membrane Model, Newtonian Case	704
9.7.1.1.1	Equations	704
9.7.1.1.2	Dimensionless Variables	706
9.7.1.1.3	Solution	707
9.7.1.2	Two-Dimensional Membrane Model: Viscoelastic Case	707
9.7.1.2.1	Dimensionless Variables	708
9.7.1.2.2	Solution	709
9.7.1.3	Two-Dimensional Membrane Model	709
9.7.2	Appendix 2: Cooling of Films in Air or Water	711
9.7.2.1	Problem Statement	711
9.7.2.2	Solution	712
9.7.2.3	Cooling of the Film in Air	712
9.7.2.3.1	Heat Transfer Coefficient by Convection	712
9.7.2.3.2	Heat Transfer Coefficient by Radiation	713
9.7.2.3.3	Cooling Calculations	714
9.7.2.3.4	Results	714
9.7.2.4	Cooling of the Film in Water	716
9.7.3	Appendix 3: Solving the Film Blowing Equations	718
9.7.3.1	Newtonian Case	718
9.7.3.1.1	Equations and Unknowns	718
9.7.3.1.2	Dimensionless Variables	719
9.7.3.1.3	Solution	721
9.7.3.2	Viscoelastic Case	722
9.7.3.2.1	Equations and Unknowns	722
9.7.3.2.2	Dimensionless Variables	723
9.7.3.2.3	Solution	724
References		725
10	Flow Instabilities	731
10.1	Extrusion Defects	731

10.1.1	Description of the Various Defects Observed in Capillary Rheometry	731
10.1.2	Extrusion Defects of Linear Polymers.	734
10.1.2.1	Sharkskin Defect.	734
10.1.2.1.1	Description	734
10.1.2.1.2	Defect Quantification	734
10.1.2.1.3	Key Parameters	737
10.1.2.1.4	Interpretation	739
10.1.2.1.5	Remedies	742
10.1.2.2	Oscillating Defect	745
10.1.2.2.1	Presentation.	745
10.1.2.2.2	Key Parameters	747
10.1.2.2.3	Bagley Corrections	748
10.1.2.2.4	Stress at the Walls.	749
10.1.2.2.5	Description of the Oscillating Cycle	750
10.1.2.2.6	Interpretation and Mechanisms	752
10.1.2.2.7	Molecular Interpretation	755
10.1.2.2.8	Example of Descriptive Model.	756
10.1.2.2.9	Remedies	758
10.1.3	Extrusion Defects of Branched Polymers	759
10.1.3.1	Description	759
10.1.3.2	Wall Shear Stress	761
10.1.3.3	Influence of Geometry	762
10.1.3.4	Interpretation	764
10.1.3.4.1	Remedies	768
10.1.4	Summary and Outlook	769
10.2	Coextrusion Defects	770
10.2.1	Investigation of Coextrusion Instabilities.	770
10.2.1.1	Influence of the Flow Configuration.	770
10.2.1.2	Analysis of the Flow within a Coextrusion Die	771
10.2.2	Modeling Coextrusion Instabilities.	774
10.2.2.1	Convective Stability Investigation.	774
10.2.2.1.1	Asymptotic Stability Analysis	775
10.2.2.1.2	Convective Stability Analysis.	776
10.2.2.2	Direct Numerical Simulation.	778
10.2.3	Conclusions	780
10.3	Calendering Defects	781
10.3.1	Different Types of Defects	781
10.3.2	Analysis of the Mattiness Defect	783
10.3.3	Analysis of the V-Shaped Defect	784
10.3.4	Analysis of the Rocket Defect.	786

10.3.5	Conclusions	788
10.4	Drawing Instabilities	789
10.4.1	Description of Drawing Instabilities	789
10.4.1.1	Example of Fiber Spinning	789
10.4.1.2	Example of the Cast-Film Process	791
10.4.1.3	Example of the Film-Blowing Process	792
10.4.1.4	Conclusions	794
10.4.2	Modeling Fiber Spinning Instability	795
10.4.2.1	Stretching of a Newtonian Fluid under Isothermal Conditions	795
10.4.2.2	Influence of Thermal Phenomena	796
10.4.2.3	Influence of Viscoelasticity	798
10.4.3	Modeling Cast-Film Instability	799
10.4.3.1	Stability of a Constant Film Width Stretching Model	799
10.4.3.2	Stability of a 1D Membrane Model Accounting for Neck-In	800
10.4.3.3	Stability of the 2D Membrane Model	802
10.4.4	Modeling Film-Blowing Instabilities	803
10.4.5	Conclusion	806
	References	806
	Notations	817
	Color Supplement	827
	Subject Index	837

Foreword to the English Edition

It was with great enthusiasm that I agreed to compose this foreword for the second edition of *Polymer Processing: Principles and Modeling* (P³M-2). In 1994, when I arrived at the Mechanical Engineering Department of the University of Wisconsin – Madison, it was Professor Tim Osswald who introduced me to teaching from the first edition of this book (P³M-1). I then taught the introductory course on polymer processing from P³M-1, twice a year, for years to come. My senior elective course classroom was well populated by students from the departments of Mechanical Engineering, Chemical Engineering, and Materials Science and Engineering. P³M-1 was a student favourite for its readability and its expert use of terms with plain meaning, wherever possible. I used this first edition until, disappointingly, it went out of print. P³M-2 expands on P³M-1 from 6 chapters to 10, and P³M-2 is reorganized, now opting to cover rheology in one consolidated second chapter rather than postponing viscoelasticity until Chapter 6. This expansion and reorganization are clever improvements. I am pleased to report that Chapter 2 retains a clear explanation of the Jaumann derivative, making Chapter 2 a gem. I see that the writing style still employs terms with plain meaning, wherever possible. Undergraduate students, the hardest to please, will enjoy this book.

Each chapter is designed pedagogically to sets students free to solve a broad class of relevant problems, as it should. For instance, Chapter 7 on injection molding equips students to solve time-unsteady processing problems, Chapter 5 on single-screw extrusion enables students to attack problems with non-obvious coordinates systems, and Chapter 8 on calendering teaches students how an apparently complicated process geometry, cleverly chosen, may yield process working equations of remarkable simplicity. In Chapter 6 on twin-screw extrusion, new to P³M-2, we enjoy Vergnes' special touch, the foremost authority on extrusion, and Chapter 8 on calendering, bears Agassant's signature, who for decades has been the foremost authority on this process. P³M-2 is a translation from the recent French fourth edition [*Mise en forme des polymères* (2014)] and, as was the case for P³M-1, P³M-2 has the readability of English first language authorship.

Our world's polymer processing industry continues to grow steadily, to employ and to govern our prosperity and quality of life. Creative polymer chemists and product designers continue to challenge plastics engineers with novel combinations of material and shape. Our need to arrive at solutions to the ensuing manufacturing problems, in a hurry, confidently, and inexpensively, more than ever, requires our plastics engineering community to be well versed in the fundamentals of plastics processing. P³M-2 addresses this need expertly by empowering plastics engineers to create knowledge about plastics processing, and thus, to fill knowledge gaps, as they arise, in our quickly evolving world of plastics manufacturing.

A. Jeffrey Giacomin, PhD, PEng, PE
Tier 1 Canada Research Chair in Rheology
Queen's University at Kingston, Canada

Preface to the Third French Edition

The viscoelastic properties of long chain molecules are quite extraordinary. Even in a highly diluted solution (100 parts per million), polyethylene oxide drastically reduces the turbulent losses of water. It also allows tubeless siphons to function, as discovered by James in Toronto, which are fascinating objects. The same for molten polymers: in very slow flows, they behave like liquids. In more rapid motions, they behave like rubber and, in flow near walls, they exhibit astonishing slip properties that we are beginning to examine at Collège de France using rather sophisticated optical techniques. All that I briefly described here has major practical implications, in particular for the processing of plastic materials. In injection molding, extrusion, or more sophisticated processes, consistently one has to force the liquid polymer to rapidly adopt preset shapes—which it does not like. Hence the many defects in the final product, such as sharkskin, which is a disaster for the manufacturer of extruded products. Plastics engineering is, therefore, a difficult art, and the authors describe here the basic notions based on extensive experiences, working directly with many manufacturers. Their approach is based mainly on principles of mechanics, but they have incorporated in their first chapters (and a few other places) a useful introduction to the physical underlying phenomena. Of course, this introduction is no substitute for basic textbooks such as that of John Ferry on viscoelasticity, or that of S. Edwards and M. Doi on the behavior of entangled chains. The first edition of this book has already been proven to be quite useful: chemical engineering communities in France and Canada have heavily relied on it. This new version, which is significantly expanded, should be of great service; I wish it great success.

P.G. de Gennes, Nobel Prize in Physics 1991
December 1995

Translated by P.J. Carreau
August 2016

Acknowledgements

Four authors of this book are or have been associated with the Centre de Mise en Forme des Matériaux (Materials Forming Center, CEMEF) of Ecole des Mines de Paris (now MINES-ParisTech).

This research center was established in 1974, and it was one of the first institutions to be established in the Sophia-Antipolis Techno-park (Alpes-Maritimes, France) in 1976. It has been associated with the Centre National de la Recherche Scientifique (CNRS) since 1979 (joint research Unit 7635). It now has nearly one hundred fifty people: professors, researchers, PhD students, advanced-master and master students, engineers, technicians, and administrative staff.

The role of CEMEF is twofold:

- Training, in the field of engineering materials and processing, of engineers, master, and PhD students. Since the beginning, nearly 450 doctoral degrees and more than 350 advanced-master's degrees were supported by the center. These graduates are now working in many industrial companies with which the center is related.
- Contribution to solving scientific and technical problems in the field of processing and forming of materials (particularly metals and polymers). The center maintains relations with the major French and European companies in the development, implementation, and use of materials.

Jean-François Agassant is an engineer from Ecole des Mines de Paris, Doctor of Science, and professor at the Ecole des Mines de Paris. He was deputy director of CEMEF (1981–2007) and director of the joint unit between MINES-ParisTech and CNRS (1989–2001). He is now responsible for the “Mechanical and Material Engineering” department and the head of MINES-ParisTech on the Sophia-Antipolis site.

Pierre Avenas is an alumnus of Ecole Polytechnique (Paris) and engineer “corps of Mines.” He initiated research on polymers at the Ecole des Mines de Paris and helped create CEMEF, of which he was director from 1974 to late 1978. After heading the industrial research department at the Ministry of Industry of France (1979–1981), he held several positions in the chemical industry, including Director of R & D chemistry of Total group until 2004.

Bruno Vergnes is an engineer from ENSTA (École nationale supérieure de techniques avancées), Doctor-engineer from Ecole des Mines de Paris, and Doctor of Science. He worked from 1981 to 2008 at CEMEF, in the research group “Viscoelastic Flows”, with J.F. Agassant and M. Vincent. He is currently director of research at MINES-ParisTech and responsible for continuous processes and rheological problems in the research unit “Polymers and Composites” at CEMEF.

Michel Vincent is an engineer from Ecole des Mines of Saint-Etienne and Doctor of engineering from Ecole des Mines de Paris. He is currently director of research at CNRS, and he is responsible within the research unit “Polymers and Composites” of CEMEF for the injection molding and reinforced polymers.

The fifth author, **Pierre Carreau**, was responsible for the translation and adaptation of the original French book into English. He graduated in chemical engineering from Ecole Polytechnique of Montreal. He obtained his PhD from the University of Wisconsin (Madison, USA). He is now professor emeritus of Ecole Polytechnique of Montreal. He was the founder of the Center on Applied Polymer Research and, more recently, of the Research Center for High Performance Polymer and Composite Systems (CREPEC). CREPEC is an interuniversity research center joining 50 of Quebec’s scientists specialized in the development of new high performance polymers and composites and their transformation and implementation process.

Both CEMEF and CREPEC have been associated for many years. Initially, under the France-Quebec collaboration program, a few joint research projects have been initiated. The first English book, published in 1991, and this revised and expanded version are major outcomes of this collaboration.

The initial French book was first published in 1982 and updated in 1986, 1996, and 2014. The second edition in 1986 was adapted and translated into English by Pierre Carreau; it was published by Hanser in 1991. The present translated version of the latest French edition is completely redesigned, both in the presentation and scope of the topics. It presents a synthesis of research and teaching approaches developed over more than thirty years in the field of processing of polymers at CEMEF.

We would like to mention all researchers, colleagues, doctoral and master’s graduates, who were or are still at CEMEF and at Polytechnique Montreal, whose work has contributed to the realization of this book: H. Alles, J.M. André, B. Arpin, G. Ausias, Ph. Barq, C. Barrès, S. Batkam, P. Beaufils, M. Bellet, N. Bennani, C. Beraudo, F. Berzin, R. Blanc, F. Boitout, R. Bouamra, C. Champin, M. Coevoet, C. Combeaud, D. Cotto, T. Coupeuz, L. Delamare, Y. Demay, F. Démé, O. Denizart, E. Deviliers, F. Dimier, T. Domenech, J.L. Dournaux, C. Dubrocq-Baritaud, R. Ducloux, V. Durand, A. Durin, M. Espy, E. Foudrinier, E. Gamache, J.F. Gobeau, S. d’Halewyn, J.M. Haudin, I. Hénaut, C. Hoareau, S. Karam, D. Kay, M. Koscher, P. Lafleur, P. Laure, M. Leboeuf, D. Le Roux, W. Lertwimolnun, O. Mahdaoui, H. Maders, R. Magnier, B. Magnin, J. Mauffrey, M. Mouazen, Ph. Mourniac, B. Neyret, I. Noé, H. Nouatin, L. Parent,

C. Peiti, S. Philipon, A. Philippe, A. Piana, E. Pichelin, A. Poitou, A. Poulesquen, S. Mighry, L. Robert, A. Rodriguez-Villa, P. Saillard, G. Schlatter, F. Schmidt, D. Silagy, L. Silva, C. Sollogoub, G. Sornberger, B. Souloumiac, J. Tayeb, J. Teixeira-Pirès, R. Valette, C. Venet, E. Wey, and J.L. Willien. Our thanks go to them and to all those with whom we had the opportunity to work, in both French and foreign universities and in industry, on topics of rheology and polymer processing.

Finally, we thank Ms. Corinne Matarasso who improved the quality of many figures.

1

Continuum Mechanics: Review of Principles

■ 1.1 Strain and Rate-of-Strain Tensor

1.1.1 Strain Tensor

1.1.1.1 Phenomenological Definitions

Phenomenological definitions of strain are first presented in the following examples.

1.1.1.1.1 Extension (or Compression)

In extension, a volume element of length l is elongated by Δl in the x direction, as illustrated by Figure 1.1. The strain can be defined, from a phenomenological point of view, as $\varepsilon = \Delta l/l$.

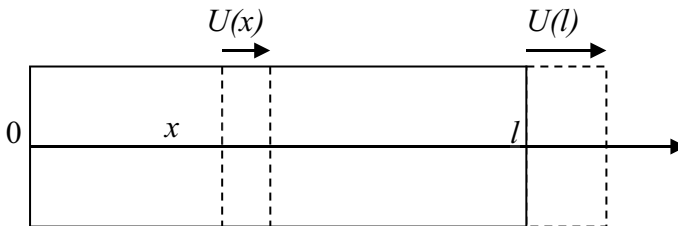


Figure 1.1 Strain in extension

For a homogeneous deformation of the volume element, the displacement U on the x -axis is $U(x) = \Delta l \frac{x}{l}$, and $\frac{dU}{dx} = \frac{\Delta l}{l}$. Hence another definition of the strain is $\varepsilon = \frac{dU}{dx}$.

1.1.1.1.2 Pure Shear

A volume element of square section $h \times h$ in the x - y plane is sheared by a value a in the x -direction, as shown in Figure 1.2. Intuitively, the strain may be defined as $\gamma = a/h$. For a homogeneous deformation of the volume element, the displacement (U, V) of point $M(x, y)$ is

$$U(y) = a \frac{y}{h}; V = 0 \quad (1.1)$$

Hence, another possible definition of the strain is $\gamma = \frac{dU}{dy}$.

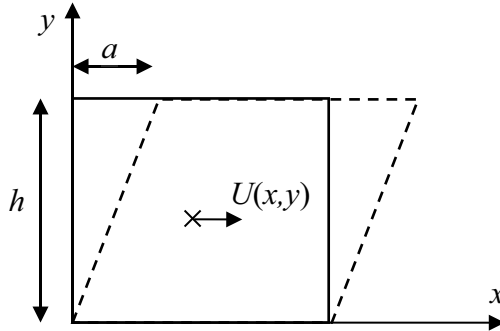


Figure 1.2 Strain in pure shear

1.1.1.2 Displacement Gradient

More generally, any strain in a continuous medium is defined through a field of the displacement vector $\mathbf{U}(x, y, z)$ with coordinates

$$U(x, y, z), \quad V(x, y, z), \quad W(x, y, z)$$

The intuitive definitions of strain make use of the derivatives of U , V , and W with respect to x , y , and z , that is, of their gradients. For a three-dimensional flow, the material can be deformed in nine different ways: three in extension (or compression) and six in shear. Therefore, it is natural to introduce the nine components of the displacement gradient tensor $\nabla \mathbf{U}$:

$$\nabla \mathbf{U} = \begin{bmatrix} \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} & \frac{\partial U}{\partial z} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \\ \frac{\partial W}{\partial x} & \frac{\partial W}{\partial y} & \frac{\partial W}{\partial z} \end{bmatrix} \quad (1.2)$$

This notion of displacement gradient applied to the two previous deformations presented in Section 1.1.1.1 leads to the following expressions:

- Extension deformation:

$$\nabla \mathbf{U} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.3)$$

- Shear deformation:

$$\nabla \mathbf{U} = \begin{bmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.4)$$

If this notion is applied to a volume element that has rotated θ degrees without being deformed, as shown in Figure 1.3, the displacement vector can be written as

$$\mathbf{U} = \begin{cases} U(x, y) = x(\cos\theta - 1) - y\sin\theta \\ V(x, y) = x\sin\theta + y(\cos\theta - 1) \end{cases} \quad (1.5)$$

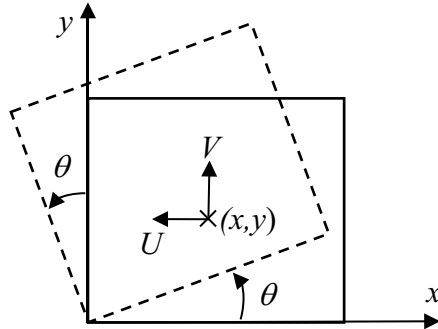


Figure 1.3 Rigid rotation

$$\text{For a very small value of } \theta: \quad \begin{aligned} U(x, y) &\approx -y\theta \\ V(x, y) &\approx x\theta \end{aligned} \quad (1.6)$$

$$\text{hence} \quad \nabla \mathbf{U} = \begin{bmatrix} 0 & -\theta & 0 \\ \theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.7)$$

It is obvious from this result that $\nabla \mathbf{U}$ cannot physically describe the strain of the material since it is not equal to zero when the material is under rigid rotation without being deformed.

1.1.1.3 Deformation or Strain Tensor $\boldsymbol{\varepsilon}$

To obtain a tensor that physically represents the local deformation, we must make the tensor $\nabla\mathbf{U}$ symmetrical, as follows:

- Write the transposed tensor (symmetry with respect to the principal diagonal); the transposed deformation tensor is

$$(\nabla\mathbf{U})^t = \begin{bmatrix} \frac{\partial U}{\partial x} & \frac{\partial V}{\partial x} & \frac{\partial W}{\partial x} \\ \frac{\partial U}{\partial y} & \frac{\partial V}{\partial y} & \frac{\partial W}{\partial y} \\ \frac{\partial U}{\partial z} & \frac{\partial V}{\partial z} & \frac{\partial W}{\partial z} \end{bmatrix} \quad (1.8)$$

- Write the half sum of the two tensors, each transposed with respect to the other:

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{U} + (\nabla\mathbf{U})^t) \quad (1.9)$$

$$\text{or } \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \quad (1.10)$$

where U_i stands for U , V , or W and x_i for x , y , or z .

Let us now reexamine the three previous cases:

- In *extension* (or compression):

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.11)$$

The deformation tensor $\boldsymbol{\varepsilon}$ is equal to the displacement gradient tensor $\nabla\mathbf{U}$.

- In *pure shear*:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} 0 & \frac{1}{2}\gamma & 0 \\ \frac{1}{2}\gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.12)$$

The tensor $\boldsymbol{\varepsilon}$ is symmetric, whereas $\nabla\mathbf{U}$ is not. We see that pure shear is physically imposed in a nonsymmetrical manner with respect to x and y ; however, the strain experienced by the material is symmetrical.

- In *rigid rotation*:

$$\boldsymbol{\epsilon} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.13)$$

The definition of $\boldsymbol{\epsilon}$ is such that the deformation is nil in rigid rotation; it is physically satisfactory, whereas the use of $\nabla \mathbf{U}$ for the deformation is not correct.

As a general result, the tensor $\boldsymbol{\epsilon}$ is always symmetrical; that is, it contains only six independent components:

- three in extension or compression: ϵ_{xx} , ϵ_{yy} , ϵ_{zz}
- three in shear: $\epsilon_{xy} = \epsilon_{yx}$, $\epsilon_{yz} = \epsilon_{zy}$, $\epsilon_{zx} = \epsilon_{xz}$

Important Remarks

(a) The definition of the tensor $\boldsymbol{\epsilon}$ used here is a simplified one. One can show rigorously that the strain tensor in a material is mathematically described by the tensor Δ (Salençon, 1988):

$$\Delta_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} + \sum_k \frac{\partial U_k}{\partial x_i} \frac{\partial U_k}{\partial x_j} \right) = \epsilon_{ij} + \frac{1}{2} \sum_k \frac{\partial U_k}{\partial x_i} \frac{\partial U_k}{\partial x_j} \quad (1.14)$$

This definition of the tensor $\boldsymbol{\epsilon}$ is valid only if the terms $\partial U_i / \partial x_j$ are small. So the expressions for the tensor written above are usable only if ϵ , γ , θ , and so on are small (typically less than 5%). This condition is not generally satisfied for the flow of polymer melts. As will be shown, in those cases, we will use the rate-of-strain tensor $\dot{\boldsymbol{\epsilon}}$.

(b) The deformation can also be described by following the homogeneous deformation of a continuum media with time. The Cauchy tensor is then used, defined by

$$\mathbf{C} = \mathbf{F} \cdot \mathbf{F}^t \text{ with } F_{ij} = \frac{\partial x_i}{\partial X_j} \quad (1.15)$$

where x_i are the coordinates at time t of a point initially at X_i , and \mathbf{F}^t is the transpose of \mathbf{F} . The inverse tensor, called the Finger tensor, will be used in Chapter 2:

$$\mathbf{C}^{-1} = \mathbf{F}^{-1} \cdot (\mathbf{F}^t)^{-1} \quad (1.16)$$

1.1.1.4 Volume Variation During Deformation

Only in extension or compression the strain may result in a variation of the volume. If l_x , l_y , l_z are the dimensions along the three axes, the volume, \mathcal{V} , is then

$$\mathcal{V} = l_x l_y l_z \Rightarrow \frac{d\mathcal{V}}{\mathcal{V}} = \frac{dl_x}{l_x} + \frac{dl_y}{l_y} + \frac{dl_z}{l_z} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} \quad (1.17)$$

1.1.2 Rate-of-Strain Tensor

For a velocity field $\mathbf{u}(x, y, z)$, the rate-of-strain tensor is defined as the limit:

$$\dot{\boldsymbol{\epsilon}} = \lim_{dt \rightarrow 0} \frac{\boldsymbol{\epsilon}_t^{t+dt} - \boldsymbol{\epsilon}_t}{dt} \quad (1.18)$$

where $\boldsymbol{\epsilon}_t^{t+dt}$ is the deformation tensor between times t and $t + dt$. However, in this time interval the displacement vector is $d\mathbf{U} = \mathbf{u} dt$. Hence,

$$\epsilon_{ij}^{t+dt} - \epsilon_{ij}^t = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) dt \quad (1.19)$$

where $u_i = (u, v, w)$ are the components of the velocity vector. The components of the rate-of-strain tensor become

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (1.20)$$

As in the case of $\boldsymbol{\epsilon}$, this tensor is symmetrical:

$$\dot{\boldsymbol{\epsilon}} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^t) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{\partial v}{\partial y} & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) & \frac{\partial w}{\partial z} \end{bmatrix} \quad (1.21)$$

The diagonal terms are elongational rates; the other terms are shear rates. They are often denoted $\dot{\alpha}$ and $\dot{\gamma}$, respectively.

Remark: Equation (1.20) is the general expression for the components of the rate-of-strain tensor, but its derivation from the expression (1.18) for the strain tensor is correct only if the deformations and the displacements are infinitely small (as in the case of a high-modulus elastic body). For a liquid material, it is not possible, in general, to make use of expression (1.19). Indeed, a liquid experiences very large deformations for which the tensor $\boldsymbol{\epsilon}$ has no physical meaning. Tensors $\mathbf{\Delta}$, \mathbf{C} , or \mathbf{C}^{-1} are used instead.

1.1.3 Continuity Equation

1.1.3.1 Mass Balance

Let us consider a volume element of fluid $dx dy dz$ (Figure 1.4). The fluid density is $\rho(x, y, z, t)$.

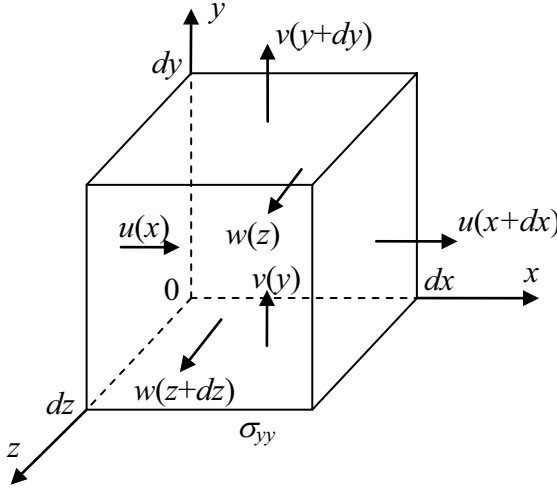


Figure 1.4 Mass balance on a cubic volume element

The variation of mass in the volume element with respect to time is $\frac{\partial \rho}{\partial t} dx dy dz$. This variation is due to a balance of mass fluxes across the faces of the volume element:

- In the x direction: $(\rho(x+dx)u(x+dx) - \rho(x)u(x)) dy dz$
- In the y direction: $(\rho(y+dy)v(y+dy) - \rho(y)v(y)) dz dx$
- In the z direction: $(\rho(z+dz)w(z+dz) - \rho(z)w(z)) dx dy$

Hence, dividing by $dx dy dz$ and taking the limits, we get

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (1.22)$$

which can be written through the definition of the divergence as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1.23)$$

This is the continuity equation.

Remark: This equation can be written using the material derivative $\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho$, leading to $\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{u} = 0$.

1.1.3.2 Incompressible Materials

For incompressible materials, ρ is a constant, and the continuity equation reduces to

$$\nabla \cdot \mathbf{u} = 0 \quad (1.24)$$

This result can be obtained from the expression for the volume variation in small deformations:

$$\frac{d\mathcal{V}}{\mathcal{V}} = \text{tr } \boldsymbol{\varepsilon} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \quad (1.25)$$

$$\text{also: } \frac{1}{\mathcal{V}} \frac{d\mathcal{V}}{dt} = \text{tr } \dot{\boldsymbol{\varepsilon}} = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \mathbf{u} \quad (1.26)$$

$$\text{It follows that } \frac{d\mathcal{V}}{dt} = 0 \Leftrightarrow \text{tr } \dot{\boldsymbol{\varepsilon}} = 0 \Leftrightarrow \nabla \cdot \mathbf{u} = 0 \quad (1.27)$$

1.1.4 Problems

1.1.4.1 Analysis of Simple Shear Flow

Simple shear flow is representative of the rate of deformation experienced in many practical situations. Homogeneous, simple planar shear flow is defined by the following velocity field:

$$u(y) = \dot{\gamma}y \left(\dot{\gamma} = \frac{U}{h} \right); \quad v = 0; \quad w = 0$$

where Ox is the direction of the velocity, Oxy is the shear plane, and planes parallel to Oxz are sheared surfaces; $\dot{\gamma}$ is the shear rate. Write down the expression for the tensor $\dot{\boldsymbol{\varepsilon}}$ for this simple planar shear flow.

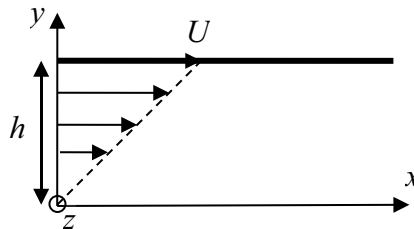


Figure 1.5 Flow between parallel plates

Solution

$$\dot{\mathbf{e}} = \begin{bmatrix} 0 & \frac{1}{2}\dot{\gamma} & 0 \\ \frac{1}{2}\dot{\gamma} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.28)$$

1.1.4.2 Study of Several Simple Shear Flows

One can assume that any flow situation is locally simple shear if, at that given point, the rate-of-strain tensor is given by the above expression (Eq. (1.28)). Then show that all the following flows, encountered in practical situations, are locally simple shear flows. Obtain in each case the directions 1, 2, 3 (equivalent to x, y, z for planar shear) and the expression of the shear rate $\dot{\gamma}$ (use the expressions of $\dot{\mathbf{e}}$ in cylindrical and spherical coordinates given in Appendix 1, see Section 1.4.1).

1.1.4.2.1 Flow between Parallel Plates (Figure 1.6)

The velocity vector components are $u(y), v = 0, w = 0$.

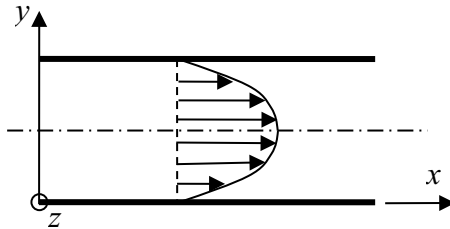


Figure 1.6 Flow between parallel plates

Solution

$$\dot{\mathbf{e}} = \begin{bmatrix} 0 & \frac{1}{2} \frac{du}{dy} & 0 \\ \frac{1}{2} \frac{du}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.29)$$

1.1.4.2.2 Flow in a Circular Tube (Figure 1.7)

The components of the velocity vector $\mathbf{u}(r, \theta, z)$ in a cylindrical frame are $u = 0, v = 0, w = w(r)$.

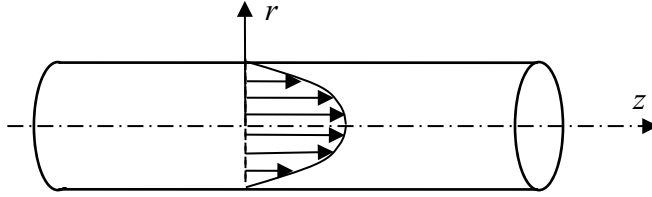


Figure 1.7 Flow in a circular tube

Solution

$$\dot{\boldsymbol{\epsilon}} = \begin{bmatrix} 0 & 0 & \frac{1}{2} \frac{dw}{dr} \\ 0 & 0 & 0 \\ \frac{1}{2} \frac{dw}{dr} & 0 & 0 \end{bmatrix} \quad (1.30)$$

Directions 1, 2, and 3 are respectively z , r , and θ . The shear rate is $\dot{\gamma} = \frac{dw}{dr}$.

1.1.4.2.3 Flow between Two Parallel Disks

The upper disk is rotating at an angular velocity Ω_0 , and the lower one is fixed (Figure 1.8). The velocity field in cylindrical coordinates has the following expression:

$$\mathbf{u}(r, \theta, z) : u = 0, v(r, z), w = 0$$

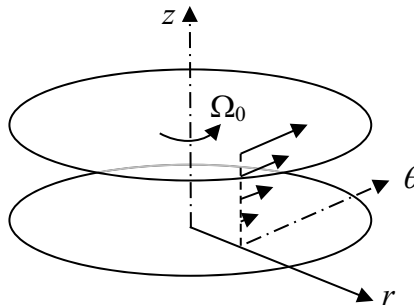


Figure 1.8 Flow between parallel disks

- Show that the tensor $\dot{\boldsymbol{\epsilon}}$ does not have the form defined in Section 1.1.4.1.
- The sheared surfaces are now assumed to be parallel to the disks and rotate at an angular velocity $\Omega(z)$. Calculate $v(r, z)$ and show that the tensor $\dot{\boldsymbol{\epsilon}}$ is a simple shear one.

Solution

(a)

$$\dot{\boldsymbol{\epsilon}} = \begin{bmatrix} 0 & \frac{1}{2} \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) & 0 \\ \frac{1}{2} \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) & 0 & \frac{1}{2} \frac{\partial v}{\partial z} \\ 0 & \frac{1}{2} \frac{\partial v}{\partial z} & 0 \end{bmatrix} \quad (1.31)$$

(b) If $v(r, z) = r\Omega(z)$, then $\frac{\partial v}{\partial r} - \frac{v}{r} = 0$ and $\dot{\boldsymbol{\epsilon}}$ is a simple shear tensor. The shear rate is $\dot{\gamma} = \frac{dv}{dz} = r \frac{d\Omega}{dz}$ and directions 1, 2, and 3 are θ , z , and r , respectively.

1.1.4.2.4 Flow between a Cone and a Plate

A cone of half angle θ_0 rotates with the angular velocity Ω_0 . The apex of the cone is on the disk, which is fixed (Figure 1.9). The sheared surfaces are assumed to be cones with the same axis and apex as the cone-and-plate system; they rotate at an angular velocity $\Omega(\theta)$.

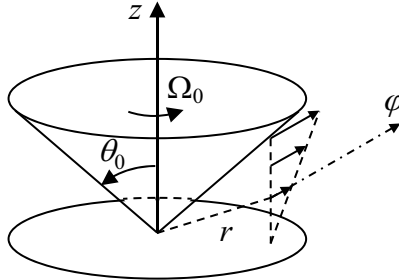


Figure 1.9 Flow in a cone-and-plate system

Solution

In spherical coordinates (r, θ, φ) , the velocity vector components are $u = 0$, $v = 0$, and $w = r \sin\theta \Omega(\theta)$.

$$\dot{\boldsymbol{\epsilon}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \sin\theta \frac{d\Omega}{d\theta} \\ 0 & \frac{1}{2} \sin\theta \frac{d\Omega}{d\theta} & 0 \end{bmatrix} \quad (1.32)$$

The shear rate is $\dot{\gamma} = \sin\theta \frac{d\Omega}{d\theta}$, and directions 1, 2, and 3 are φ , θ , and r , respectively.

1.1.4.2.5 Couette Flow

A fluid is sheared between the inner cylinder of radius R_1 rotating at the angular velocity Ω_0 and the outer fixed cylinder of radius R_2 (Figure 1.10). The components of the velocity vector $\mathbf{u}(r, \theta, z)$ in cylindrical coordinates are $u = 0$, $v(r)$, and $w = 0$.

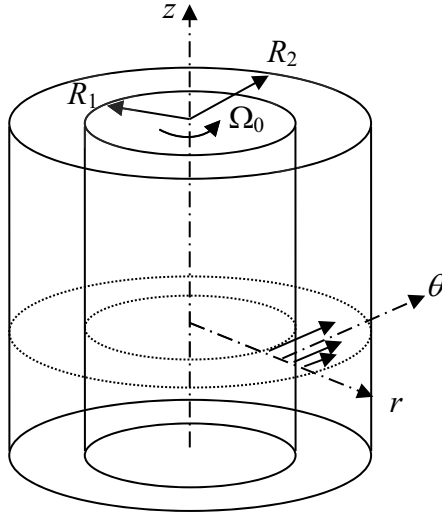


Figure 1.10 Couette flow

Solution

$$\dot{\mathbf{e}} = \begin{bmatrix} 0 & \frac{1}{2} \left(\frac{dv}{dr} - \frac{v}{r} \right) & 0 \\ \frac{1}{2} \left(\frac{dv}{dr} - \frac{v}{r} \right) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.33)$$

The shear rate is $\dot{\gamma} = \frac{dv}{dr} - \frac{v}{r}$, and directions 1, 2, and 3 are θ , r , and z , respectively.

1.1.4.3 Pure Elongational Flow

A flow is purely elongational or extensional at a given point if the rate-of-strain tensor at this point has only nonzero components on the diagonal.

1.1.4.3.1 Simple Elongation

An incompressible parallelepiped specimen of square section is stretched in direction x (Figure 1.11). Then $\dot{\alpha} = \frac{1}{l} \frac{dl}{dt}$ is called the elongation rate in the x -direction.

Write down the expression of $\dot{\mathbf{e}}$.

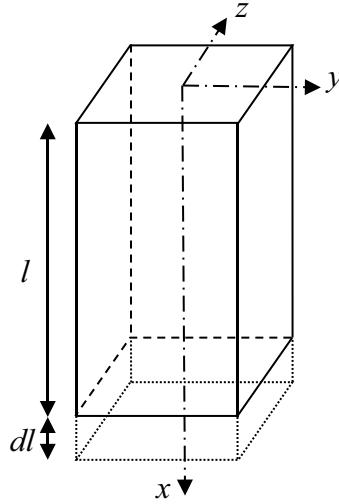


Figure 1.11 Deformation of a specimen in elongation

Solution

Assuming a homogeneous deformation, the velocity vector is $\mathbf{u} = (u(x), v(y), w(z))$ and

$$\frac{du}{dx} = \dot{\alpha} = \frac{1}{l} \frac{dl}{dt} \quad (1.34)$$

The sample section remains square during the deformation, so $\frac{dv}{dy} = \frac{dw}{dz}$. Incompressibility implies $\dot{\alpha} + 2\frac{dv}{dy} = 0$. Therefore, $\frac{dv}{dy} = \frac{dw}{dz} = -\frac{\dot{\alpha}}{2}$ and

$$\dot{\boldsymbol{\varepsilon}} = \begin{bmatrix} \dot{\alpha} & 0 & 0 \\ 0 & -\frac{\dot{\alpha}}{2} & 0 \\ 0 & 0 & -\frac{\dot{\alpha}}{2} \end{bmatrix} \quad (1.35)$$

1.1.4.3.2 Biaxial Stretching: Bubble Inflation

The inflation of a bubble of radius R and thickness e small compared to R is considered in Figure 1.12.

- Write the rate-of-strain components in the r, θ, φ directions.
- Write the continuity equation for an incompressible material and integrate it.
- Show the equivalence between the continuity equation and the volume conservation.

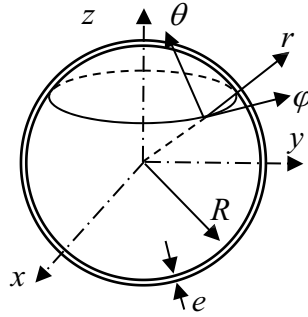


Figure 1.12 Bubble inflation

Solution

(a) The bubble is assumed to remain spherical and to deform homogeneously so that the shear components are zero. The rate-of-strain components are as follows:

- In the thickness (r) direction: $\dot{\epsilon}_{rr} = \frac{1}{e} \frac{de}{dt}$

- In the θ -direction: $\dot{\epsilon}_{\theta\theta} = \frac{1}{2\pi R} \frac{d(2\pi R)}{dt} = \frac{1}{R} \frac{dR}{dt}$

- In the φ -direction: $\dot{\epsilon}_{\varphi\varphi} = \frac{1}{2\pi R \sin\theta} \frac{d(2\pi R \sin\theta)}{dt} = \frac{1}{R} \frac{dR}{dt}$

(b) For an incompressible material, $\frac{1}{e} \frac{de}{dt} + \frac{2}{R} \frac{dR}{dt} = 0$, which can be integrated to obtain $R^2 e = \text{cst}$.

(c) This is equivalent to the global volume conservation: $4\pi R^2 e = 4\pi R_0^2 e_0$.

■ 1.2 Stresses and Force Balances

1.2.1 Stress Tensor

1.2.1.1 Phenomenological Definitions

1.2.1.1.1 Extension (or Compression) (Figure 1.13)

An extension force applied on a cylinder of section S induces a normal stress $\sigma_n = F/S$.

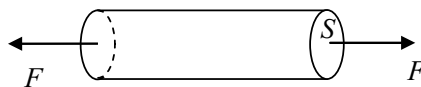


Figure 1.13 Stress in extension

1.2.1.1.2 Simple Shear (Figure 1.14)

A force tangentially applied to a surface S yields a shear stress $\tau = F/S$.

The units of the stresses are those of pressure: pascals (Pa).

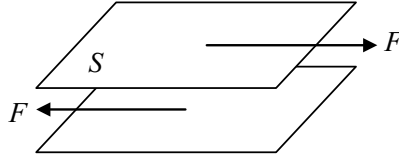


Figure 1.14 Stress in simple shear

1.2.1.2 Stress Vector

Let us consider, in a more general situation, a surface element dS in a continuum. The part of the continuum located on one side of dS exerts on the other part a force $d\mathbf{F}$. As the interactions between both parts of the continuum are at small distances, the stress vector \mathbf{T} at a point O on this surface is defined as the limit:

$$\mathbf{T} = \lim_{dS \rightarrow 0} \frac{d\mathbf{F}}{dS} \quad (1.36)$$

At point O , the normal to the surface is defined by the unit vector, \mathbf{n} , in the outward direction, as illustrated in Figure 1.15.

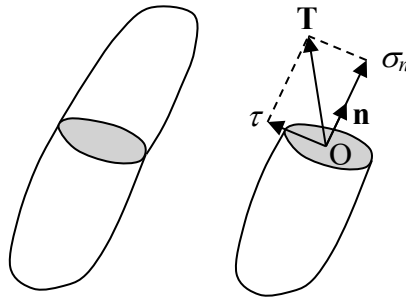


Figure 1.15 Stress applied to a surface element

The stress components can be obtained from projections of the stress vector:

- Projection on \mathbf{n} : $\sigma_n = \mathbf{T} \cdot \mathbf{n}$
where σ_n is the normal stress (in extension, $\sigma_n > 0$; in compression, $\sigma_n < 0$).
- Projection on the surface: τ is the shear stress.

1.2.1.3 Stress Tensor

The stress vector cannot characterize the state of stresses at a given point since it is a function of the orientation of the surface element, that is, of \mathbf{n} . Thus, a tensile force induces a stress on a surface element perpendicular to the orientation of the force, but it induces no stress on a parallel surface element (Figure 1.16).

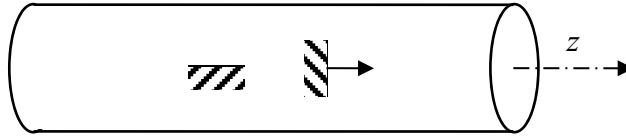


Figure 1.16 Stress vector and surface orientation

The state of stresses is in fact characterized by the relation between \mathbf{T} and \mathbf{n} and, as we will see, this relation is tensorial. Let us consider an elementary tetrahedron $OABC$ along the axes $Oxyz$ (Figure 1.17): the x , y , and z components of the unit normal vector to the ABC plane are the ratios of the surfaces OAB , OBC , and OCA to ABC :

$$n_x = \frac{OBC}{ABC} \quad n_y = \frac{OCA}{ABC} \quad n_z = \frac{OAB}{ABC}$$

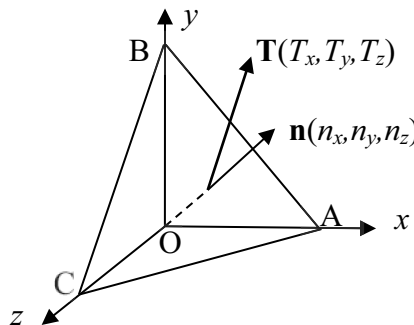


Figure 1.17 Stresses exerted on an elementary tetrahedron

Let us define the components of the stress tensor in the following table:

Projection on	of the stress vector exerted on the face normal to		
	Ox	Oy	Oz
Ox	σ_{xx}	σ_{xy}	σ_{xz}
Oy	σ_{yx}	σ_{yy}	σ_{yz}
Oz	σ_{zx}	σ_{zy}	σ_{zz}

The net surface forces acting along the three directions of the axes are as follows:

$$T_x(ABC) - \sigma_{xx}(OBC) - \sigma_{xy}(OAC) - \sigma_{xz}(OAB)$$

$$T_y(ABC) - \sigma_{yx}(OBC) - \sigma_{yy}(OAC) - \sigma_{yz}(OAB)$$

$$T_z(ABC) - \sigma_{zx}(OBC) - \sigma_{zy}(OAC) - \sigma_{zz}(OAB)$$

with OA , OB , OC being of the order of d ; the surfaces OAB , OBC , and OCA are of the order of d^2 ; and the volume $OABC$ is of the order of d^3 . The surface forces are of the order of Td^2 and the volume forces of the order of Fd^3 (e.g., $F = \rho g$ for the gravitational force per unit volume).

When the dimension d of the tetrahedron tends to zero, the volume forces become negligible compared with the surface forces, and the net forces, as expressed above, are equal to zero. Hence, in terms of the components of \mathbf{n} :

$$\begin{aligned} T_x &= \sigma_{xx}n_x + \sigma_{xy}n_y + \sigma_{xz}n_z \\ T_y &= \sigma_{yx}n_x + \sigma_{yy}n_y + \sigma_{yz}n_z \\ T_z &= \sigma_{zx}n_x + \sigma_{zy}n_y + \sigma_{zz}n_z \end{aligned} \quad (1.37)$$

This result can be written in tensorial notation as

$$\mathbf{T} = \boldsymbol{\sigma} \cdot \mathbf{n} \quad (1.38)$$

where $\boldsymbol{\sigma}$ is the stress tensor, which contains three normal components and six shear components defined for the three axes. As in the case of the strain, the state of the stresses is described by a tensor.

1.2.1.4 Isotropic Stress or Hydrostatic Pressure

The hydrostatic pressure translates into a stress vector that is in the direction of \mathbf{n} for any orientation of the surface:

$$\mathbf{T} = -p\mathbf{n} \quad (1.39)$$

The corresponding tensor is proportional to the unit tensor \mathbf{I} :

$$\boldsymbol{\sigma} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix} = -p\mathbf{I} \quad (1.40)$$

1.2.1.5 Deviatoric Stress Tensor

For any general state of stresses, the pressure can be defined in terms of the trace of the stress tensor as

$$p = -\frac{1}{3} \text{tr } \boldsymbol{\sigma} = -\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} \quad (1.41)$$

The pressure is independent of the axes since the trace of the stress tensor is an invariant (see Appendix 2, see Section 1.4.2). It could be positive (compressive state) or relatively negative (extensive state, possibly leading to cavitation problems in a liquid). The stress tensor can be written as a sum of two terms, the pressure term and a traceless stress term, called the deviatoric stress tensor $\boldsymbol{\sigma}'$:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\sigma}' \quad (1.42)$$

Examples

- *Uniaxial extension* (or compression):

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow p = -\frac{\sigma_{11}}{3}, \quad \boldsymbol{\sigma}' = \begin{bmatrix} \frac{2\sigma_{11}}{3} & 0 & 0 \\ 0 & -\frac{\sigma_{11}}{3} & 0 \\ 0 & 0 & -\frac{\sigma_{11}}{3} \end{bmatrix} \quad (1.43)$$

- *Simple shear* under a hydrostatic pressure p :

$$\boldsymbol{\sigma} = \begin{bmatrix} -p & \tau & 0 \\ \tau & -p & 0 \\ 0 & 0 & -p \end{bmatrix} \Rightarrow \boldsymbol{\sigma}' = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.44)$$

More generally, we will see that the stress tensor can be decomposed into an isotropic arbitrary part denoted as $p'\mathbf{I}$, and a tensor called the extra-stress tensor $\boldsymbol{\sigma}'$. The expressions of the constitutive equations in Chapter 2 will use either the deviatoric part of the stress tensor $\boldsymbol{\sigma}'$ for viscous behaviors or the extra-stress tensor $\boldsymbol{\sigma}'$ for viscoelastic behaviors (in this case, $\boldsymbol{\sigma}'$ is no longer a deviator, and p' is not the hydrostatic pressure).

1.2.2 Equation of Motion

1.2.2.1 Force Balances

Considering an elementary volume of material with a characteristic dimension d :

- The surface forces are of the order of d^2 , but the definition of the stress tensor is such that their contribution to a force balance is nil.
- The volume forces (gravity, inertia) are of the order of d^3 , and they must balance the derivatives of the surface forces, which are also of the order of d^3 .

We will write that the resultant force is nil (Figure 1.18).

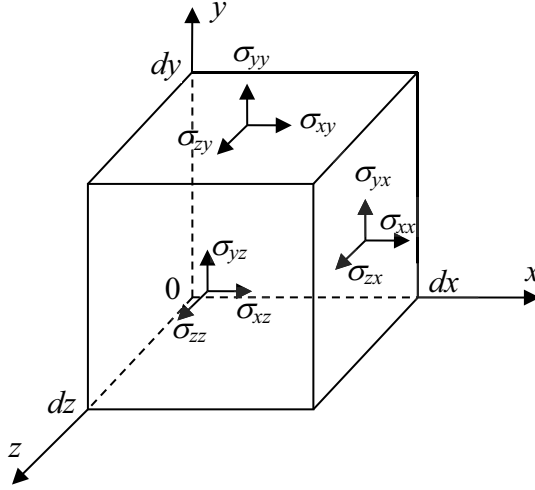


Figure 1.18 Balance of forces exerted on a volume element

The forces acting on a volume element $dx dy dz$ are the following:

- The mass force (generally gravity): $\mathbf{F} dx dy dz$
- The inertial force: $\rho \gamma dx dy dz = \rho (d\mathbf{u}/dt) dx dy dz$
- The net surface force exerted by the surroundings in the x -direction:

$$\left[\sigma_{xx}(x+dx) - \sigma_{xx}(x) \right] dydz + \left[\sigma_{xy}(y+dy) - \sigma_{xy}(y) \right] dzdx + \left[\sigma_{xz}(z+dz) - \sigma_{xz}(z) \right] dxdy$$
 and similar terms for the y and z -directions.

Dividing by $dx dy dz$ and taking the limits, we obtain for the x , y , and z components:

$$\begin{aligned} F_x - \rho \gamma_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\ F_y - \rho \gamma_y + \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\ F_z - \rho \gamma_z + \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= 0 \end{aligned} \quad (1.45)$$

The derivatives of σ_{ij} are the components of a vector, which is the divergence of the tensor $\boldsymbol{\sigma}$. Equation (1.45) may be written as

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F} - \rho \boldsymbol{\gamma} = 0 \quad (1.46)$$

This is the equation of motion, also called the dynamic equilibrium. It is often convenient to express the stress tensor as the sum of the pressure and the deviatoric stress:

$$-\nabla p + \nabla \cdot \boldsymbol{\sigma}' + \mathbf{F} - \rho \boldsymbol{\gamma} = 0 \quad (1.47)$$

1.2.2.2 Torque Balances

Let us consider a small volume element of linear dimension d ; the mass forces of the order of d^3 induce torques of the order of d^4 . There is no mass torque, which would result in torques of the order of d^3 (as in the case of a magnetic medium). Finally, the surface forces of the order of d^2 induce torques of the order of d^3 , so only the net torque resulting from these forces must be equal to zero.

If we consider the moments about the z -axis (Figure 1.19), only the shear stresses σ_{xy} and σ_{yx} on the upper (U) and lateral (L) surfaces of the element $dx dy dz$ lead to torques. They are obtained by taking the following vector products:

$$\sigma_{xy} : \begin{pmatrix} 0 \\ dy \\ 0 \end{pmatrix} \times \begin{pmatrix} \sigma_{xy} dx dz \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\sigma_{xy} dx dy dz \end{pmatrix} \quad (1.48)$$

$$\sigma_{yx} : \begin{pmatrix} dx \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ \sigma_{yx} dy dz \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sigma_{yx} dx dy dz \end{pmatrix} \quad (1.49)$$

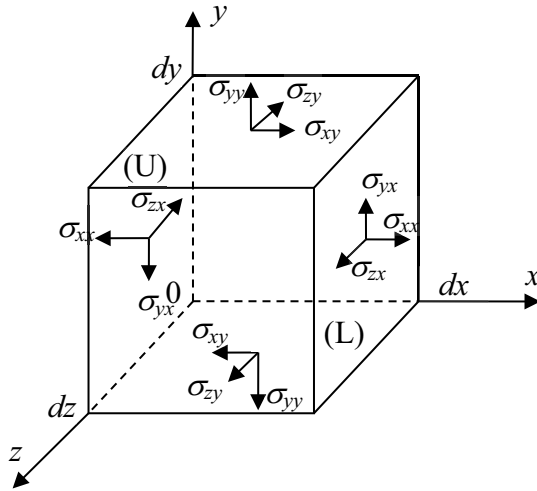


Figure 1.19 Torque balance on a volume element

A torque balance, in the absence of a mass torque, yields $\sigma_{xy} = \sigma_{yx}$. In a similar way, $\sigma_{yz} = \sigma_{zy}$ and $\sigma_{zx} = \sigma_{xz}$. The absence of a volume torque then implies the symmetry of the stress tensor. Therefore, as for the strain tensor ϵ , the stress tensor has only six independent components (three normal and three shear components).

Subject Index

Symbol

3D calculations 290,
401, 459, 468, 489,
544

A

activation energy 115,
142
adiabatic 185
adiabatic regime 205,
211, 215, 226, 227
air ring 661
approximation methods
257
Arrhenius equation 115,
267, 674, 720
asymptotic stability 775
average residence time
345, 475

B

Bagley corrections 110,
119, 748
barrier screws 336
Barr screw 339
biaxial extensional
viscosity 641
biaxial stretching 640,
663, 692
Bingham model 68
Biot number 201, 712

birefringence 136, 739,
764
blowing pressure 681,
690, 694
blow molding 679, 681
blow-up ratio 663, 664,
721, 792, 804
Brewster coefficient
136, 294
Brinkman number 204,
214, 329
Bubble geometry 675
bulk temperature 208

C

calender 589
– bank 591, 614, 784
calendering 587
– defects 781
Cameron 213
capillary number 482
capillary rheometer 108
Carreau model 50
Carreau–Yasuda model
52, 69
cast film 642, 791, 799
Cauchy tensor 5
C-chamber 435, 443,
452
centerline distance 438
chaotic defect 732, 767

characteristic curve
355, 455
chill roll 647
closure approximation 66
coalescence 484
coat-hanger die 380,
392, 405
coextrusion 408, 416
– defects 770
Cogswell method 135,
767
coinjection 567
Cole–Cole plot 128
complex modulus 80, 126
complex viscosity 69,
80, 128
compounding 488
compressibility 549,
553, 751, 756
compression ratio 335
compression zone 304,
333, 348, 373
cone-and-plate rheometer
123, 164
confined flows 255
consistency 49
Constitutive equation
35, 52, 68, 92, 252, 253
continuity equation 7
convected derivative 86,
158

- convection 196
 convective stability 774, 776
 cooling 190, 525, 558
 – of films 711
 – ring 669
 Couette flow 12, 73, 168, 307
 Couette Flow 44, 60, 101
 Coulomb's law 310
 counterpressure flow 344
 Cox–Merz rule 128
 critical draw ratio 789, 795, 798
 critical shear rate 737, 752, 754, 765, 768
 critical shear stress 784
 critical stress 737, 742, 749, 754, 755, 767
 Cross model 50
 crystallization temperature 529, 552
 cumulative strain 492
- D**
- damping function 93
 Deborah number 88, 254, 630, 652, 674, 709, 798
 deformation or strain tensor 4
 delay zone 322, 324
 deviatoric stress tensor 17
 direct numerical simulation 778
 dispersive mixing 488, 492
 dissipated power 179
 distributive mixing 488, 489
 dog-bone defect 642, 802
- drawing force 634, 665, 666
 drawing instabilities 789
 draw ratio 619, 621, 642, 664, 789
 draw resonance 789
 dynamic equilibrium 19, 26, 28
 dynamic mixer 359
- E**
- eigenvalue 776, 796, 804
 Einstein equation 61
 elastic dumbbell 89, 149
 elongational rates 6
 elongational rheometer 132
 elongational viscosity 34, 87, 103, 131
 emissivity 188, 243, 636, 713
 encapsulation 409
 energy balance 634, 655
 energy balance equation 606
 energy equation 182, 183, 184, 252, 290
 entanglement 50, 89, 739, 741, 752, 755
 enthalpy of crystallization 184
 equation of motion 18
 equilibrium regime 205, 207, 215
 exit pressure 120
 extrudate swell 72, 83
 extrusion blow molding 679, 681
 extrusion defects 731
 Eyring theory 142
- F**
- feedblock 416
 feeding 309
 – zone 303
 fiber 63, 569
 – spinning 619, 789, 795
 filled polymers 60
 filling 522, 526
 – ratio 437, 477
 film-blowing 661, 792, 803
 – die 378, 383, 408
 film shrinkage 655
 finger strain tensor 93
 finite difference methods 285
 finite element 611, 614, 693
 finite elements method 283, 287
 fixed-point method 284
 flat die 380, 392, 416
 flight angle 304, 440
 flow birefringence 294
 force balance 252
 forced convection 188, 232, 239, 242, 635, 661, 669, 712
 fountain flow 534
 Fourier's law 178
 free convection 188, 232, 235, 237
 free surface flows 255
 free volume 148
 freezing line 671, 674
 – height 662, 722
 frequency sweep 126
 friction coefficient 310, 324
 Froude number 38

- G**
Galerkin method 283, 290
gas-assisted injection molding 564
glass transition 116
Graetz number 213
Grashof number 235
grooved barrel 319
- H**
heat capacity 181, 189
heat flux 178
heat penetration thickness 193
heat transfer boundary conditions 256
heat transfer coefficient 185, 200, 214, 232, 237, 242, 246, 460, 471, 478, 635, 712
Hele-Shaw Approximations 260
Hele-Shaw equation 262, 282, 541, 553
helical defect 732, 734, 759
Herschel-Bulkley model 68
holding phase 523, 548
hydrodynamics bearings 272
hydrostatic pressure 17
- I**
incompressible materials 8
inflation time 683
injection cycle 522, 524
injection-molding machine 521
interface instability 773
interfacial tension 482
internal energy 177, 690
internal pressure 666, 680
interpenetration zone 438, 442, 454
intrinsic viscosity 62
iterative method 285, 293
- J**
Jauman derivative 94, 160
Jeffery equation 65
Johnson and Segalman model 94
- K**
Kelvin-Voigt model 76
kinematics boundary conditions 255
kinematic viscosity 36
kneading disk 436, 447, 461, 508
Krieger-Dougherty equation 62, 451
- L**
laser doppler velocimetry 140, 743, 753
left-handed screw element 444, 447, 453, 457, 461
length stretch 489
level-set 544
linear domain 126
linear stability 795, 800, 804
linear viscoelasticity 75, 108, 132
lodge model 92
longitudinal flow 343
loss modulus 80
lubrication approximations 259, 295, 297, 540, 553, 592, 594
- M**
Maillefer screw 338
mass balance 252
master curve 117, 128
material derivative 177
matteness defect 781, 783
Maxwell model 75, 85, 95
mechanical-thermal coupling 220
melt fracture 731
melting mechanism 323, 368
melting model 328, 335
melting rate 328
melting zone 303, 322, 437, 447
melt pool 322, 325, 366, 447
membrane 681, 686, 697
- hypothesis 673
- model 644, 646
memory function 93
meshing 287
mixing elements 359, 460
molecular weight 89, 144, 503, 737, 742, 747, 750, 755, 758
Mooney method 122, 761
multicavity mold 523, 575

N

nanocomposite 70, 492
 Navier–Stokes equation
 26, 28, 38
 neck-in 642, 800
 Newtonian behavior 33, 52
 Newtonian plateau 48
 Newton method 285
 no-flow temperature 529
 normal stress difference
 81, 120, 124, 130, 767,
 770
 Nusselt number 183,
 201, 214, 233, 235,
 240, 265, 353, 460,
 478, 606, 635, 712

O

Oldroyd-B model 93, 693
 Oldroyd derivative 86, 159
 optimization 357, 364,
 506
 orientation tensor 64, 94
 oscillating defect 732,
 745, 769
 oscillatory shear 126,
 130, 494
 overheating 359, 371,
 388, 390, 395

P

packing 524, 548, 549
 pancake die 409
 parallel-plate rheometer
 130
 parison 679, 686
 particulate models 320
 Peclet number 191, 206,
 290
 Phan-Thien Tanner model
 94

phase angle 126
 physical properties
 – of air 238
 – of water 238
 pipe die 379, 388
 polymer blends 481
 polymerization 503, 507
 polymer processing aids
 742
 pom-pom model 91, 293
 postextrusion calendaring
 610
 Power law 49, 52
 – index 49, 54, 393
 Prandtl number 235,
 239, 713
 preform 680, 695
 pressure-dependent
 coefficient 118
 pressure flow 40, 53,
 56, 71, 168
 pressure hole 120
 pressure oscillations
 745, 750
 principal stress difference
 294
 profile die 381, 399
 pumping zone 303, 339,
 364, 373
 pure shear 2
 PVT 524, 551

R

Rabinowitsch correction
 111, 162
 radiation 188, 242
 radiative heat transfer 636
 rate-of-strain tensor 6,
 26, 27
 Rayleigh bearing 273,
 275, 344

Rayleigh instability 482
 Rayleigh number 234
 reactive extrusion 499
 relaxation time 74, 88,
 90, 125, 129
 reptation 90
 residence time distribution
 345, 473
 residual stresses 558
 resistances 186
 restrictive elements 437
 Reynolds bearing 273,
 277
 Reynolds equation 37,
 260, 277, 348, 388,
 594, 602, 606
 Rheo-optics 135
 right-handed screw element
 437, 452, 457
 rocket defect 783, 786
 roll bending 590, 608
 roller bearing 274, 592
 Rouse model 89

S

scale-up 506
 screw pitch 304, 440
 separating force 597, 608
 shape factor 64, 347, 455
 sharkskin defect
 732–734, 769
 shear rates 6
 shear-thinning 48, 50,
 601, 603
 shift factor 114, 116
 shooting method 629, 709
 shrinkage 525, 548,
 554, 558
 simple shear 8, 24, 33,
 35, 39, 44, 53, 55, 59,
 81, 95, 168

- single-screw extruder 303
slab method 281, 637
slender body theory 263
slender thread
 approximation 623
slip velocity 599
slit die rheometer 119
solids conveying zone 445
specific heat 181
specific mechanical energy
 (SME) 450, 495
spread height 592, 594,
 603
staggering angle 466, 468
stanton number 797
static mixer 359
Stefan-Boltzmann
 constant 188, 242
stick-slip 732, 733, 745,
 752, 755
storage modulus 80
strain 1
 – hardening 132
 – recovery 74, 78
stream function 614
streamlines 597
stress 14
 – relaxation 78
 – retardation 74, 78
 – tensor 16
 – vector 15
stretch blow molding
 680, 692
stretching force 623,
 630, 632
suspension 61
- T**
Tait 552
T-die 403
temperature field 656
thermal conductivity 178
thermal contact resistance
 206, 533, 554
thermal diffusivity 189,
 191
thermal effusivity 189,
 195
thermal regime 204
thickness distribution 696
thickness recovery 592,
 612
thick shell 681
thin flow 540, 553
thin layer flows 280
thin-shell assumption
 646, 653
three-layer flow 423
time-temperature
 superposition 114, 127
transition regime 205, 213
transverse flow 341
Trouton behavior 36, 132
Trouton equation 264, 628
twin-screw extruder 433
two-layer flow 411, 420,
 425
two-stage extruder 359
- U**
unattainable zone 630,
 651, 674, 801
uniaxial extension 36
- V**
velocity-gradient tensor
 25, 27
velocity profiles 638
viscoelastic computations
 292
viscometric functions
 81, 86, 121, 165
viscosity 33, 34, 36, 48,
 109, 119, 142
viscous dissipation 180,
 604, 636
volume defects 733,
 759, 769
vorticity 615
V-shaped defect 782,
 784
- W**
wall slip 121, 741–743,
 749, 754, 761, 768
water-assisted injection
 molding 566
weight-averaged total
 strain 346
Weissenberg effect 73,
 82
Weissenberg number 88,
 254, 778, 780
wire-coating die 381,
 395
WLF equation 116
- Y**
yield stress 68, 494