HANSER

The Complete Part Design Handbook

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For Injection Molding of Thermoplastics

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Table 2-5 Flat Circular Plate Equations, Part I

W = Concentrated load (lb); w = Unit load (psi); M = Moment (in-lb/in); δ = Deflection (in); θ = Change in slope (radians); E = Modulus of elasticity (psi); H = Deflection factor (in.); υ = Poisson's ratio; σ = Stress (psi); t = Wall thickness (in); a = Outer radius (in); b = Inner radius (in); d = Shaft radius; r_0 = Radius of load (in); K = Plate constant

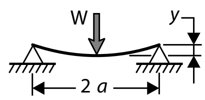
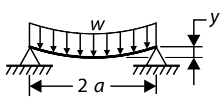
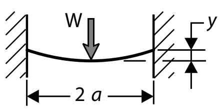
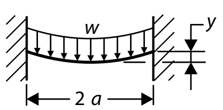
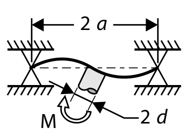
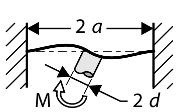
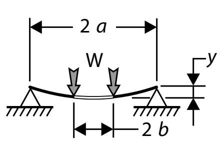
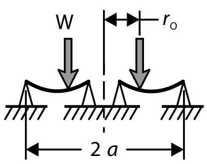
Case Type		Stress and Deflection Equations (Constant Thickness)
Concentrate Center Load Edge Simply Supported		$\sigma = \frac{3W(1+v)}{2\pi t^2}\left(\frac{1}{v+1} + \log\frac{a}{r_0} - \frac{1 - v r_0^2}{v\, 4 a^2}\right)$ $\delta_{Max.} = \frac{3 W a^2 (1-v)(3+v)}{4\pi E t^3}$
Uniform Distribute Load Edge Simply Supported		$\sigma_{Max.} = \frac{3w(3+v)}{8}\left(\frac{a^2}{t^2}\right)$ $\delta_{Max.} = \frac{3 w a^4 (1-v)(5+v)}{16 E t^3}$
Concentrated Center Load Outer Edge Fixed		For $a > r_0$ $\sigma_{Max.} = \frac{3W(1+v)}{2\pi t^2}\left(\log\frac{a}{r_0} + \frac{r_0^2}{4a^2}\right)$ $\delta_{Max.} = \frac{3 W a^2 (1-v^2)}{4\pi E t^3}$
Uniformly Distributed Load Outer Edge Fixed		$\sigma_{Max.} = \frac{3 w a^2}{4 t^2}$ $\delta_{Max.} = \frac{3 w a^4 (1-v^2)}{16 E t^3}$ $\delta_{Max.} = H\frac{3 w a^4 (1-v^2)}{16 E t^3}$ For thicker flat circular plates having $(t / a = 0.1)$, multiply the deflection equation by the constant (H), where *$H = 1 + 5.72\,(t / a)^2$.
Central Couple Outer Edge Simply Supported		$K = \frac{0.49 a^2}{(d + 0.7a)^2}$ $\sigma_{Max.} = \frac{3M}{4\pi d t^2}\left[1 + (v+1)\log\frac{2(a-d)}{K a}\right]$
Central Couple Outer Edge Fixed		$K = \frac{0.10 a^2}{(d + 0.28a)^2}$ $\sigma_{Max.} = \frac{3M}{4\pi d t^2}\left[1 + (v+1)\log\frac{2(0.45a-d)}{0.45 K a}\right]$
Radial Center Load Edge Simply Supported		$\sigma_{Max.} = \frac{3 W v}{2\pi t^2}\left[\frac{2a^2\left(\frac{1}{v}+1\right)}{(a^2+b^2)}\log\frac{a}{b} + \left(\frac{1}{\mu} - 1\right)\right]$ $\delta_{Max.} = \frac{3 W v^2\left(\frac{1}{v^2}-1\right)}{4\pi E t^3}\left[\frac{(a^2-b^2)\left(\frac{3}{v}+1\right)}{\left(\frac{1}{v}+1\right)} + \frac{4a^2b^2\left(\frac{1}{v}+1\right)}{\left(\frac{1}{v-1}\right)(a^2-b^2)}\left(\log\frac{a}{b}\right)^2\right]$

Table 2-5 Flat Circular Plate Equations, Part II

W = Concentrated load (lb); w = Unit load (psi); M = Moment (in-lb); δ = Deflection (in); θ = Angular change (rad.); Q = shear (lb/in); E = Modulus (psi); υ = Poisson's ratio; $\sigma = 6M/t^2$ (psi); t = Wall thickness (in); a = Outer radius (in); r_0 = Radius of load (in); $D = E\,t^3 / 12\,(1 - \nu^2)$; N = Equivalent radius (in); K, C, L, G = Constants (ratio-dependent)

Case Type | **Boundary Values** | **Special Cases**

Outer and Inner Edge Simply Supported; Central Radial Load



$$\delta_b = 0,\quad M_{rb} = 0,\quad \delta_a = 0,\quad M_{ra} = 0$$

$$\theta_b = \frac{-K_{\theta_b}\,W\,a^2}{D}$$

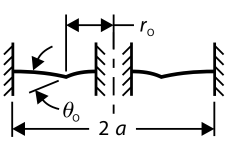
$$Q_b = K_{Qb}\,W$$

$$\theta_a = \theta_b\,C_4 + Q_b\,\frac{a^2}{D}\,C_6 - \frac{W\,a^2}{D}\,L_6$$

$$Q_a = Q_b\,\frac{b}{a} - \frac{W\,r_o}{a}$$

b/a	0.10		0.50		0.70
r_o/a	0.50	0.70	0.70	0.90	0.90
$K\delta_{Max.}$	–0.0102	–0.0113	–0.0023	–0.0017	–0.0005
$K\theta_a$	0.0278	0.0388	0.0120	0.0122	0.0055
$K\theta_b$	–0.0444	–0.0420	–0.0165	–0.0098	–0.0048
K_{Mrb}	–0.4043	–0.3819	–0.0301	–0.0178	–0.0063
K_{Mro}	0.1629	0.1689	0.1161	0.0788	0.0662
K_{Qb}	2.9405	2.4779	0.8114	0.3376	0.4145

Outer & Inner Edges Fixed; Change in Slope



$$\delta_b = 0,\quad \theta_b = 0,\quad \delta_a = 0,\quad \theta_a = 0$$

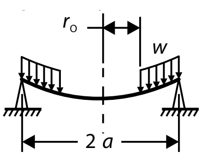
$$M_{rb} = \frac{K_{Mrb}\,\theta_o\,D}{a};\quad Q_a = Q_b\,\frac{b}{a}$$

$$Q_b = \frac{K_{Qb}\,\theta_o\,D}{a^2};\quad \delta_{Max.} = K\,\delta_o\,\theta\,a$$

$$M_{ra} = M_{rb}\,C_8 + Q_b\,a\,C_9 + \theta_o\,\frac{D}{a}\,L_7$$

b/a	0.10		0.50		0.70
r_o/a	0.50	0.70	0.70	0.90	0.90
$K\delta_o$	–0.1071	–0.0795	–0.0586	–0.0240	–0.0290
K_{Mrb}	–2.0540	1.1868	–3.5685	2.4702	0.3122
K_{Mra}	–0.6751	–1.7429	–0.8988	–5.0320	–6.3013
K_{Qb}	–0.0915	–17.0670	4.8176	–23.8910	–29.6041

Uniform Distributed Load; Edge Simply Supported



$$\delta_a = 0,\quad M_{ra} = 0;\quad M_C = w\,a^2\,G_{17}$$

$$\delta_C = \frac{-W\,a^4}{2\,D}\left(\frac{G_{17}}{1+\upsilon} - 2\,G_{11}\right)$$

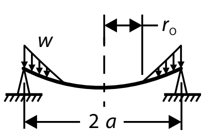
$$\theta_a = \frac{w}{8\,D\,a\,(1+\upsilon)}\,(a^2 - r_o^2)^2$$

r_o/a	0.00	0.20	0.40	0.60	0.80
$K\delta_C$	–0.0637	–0.0576	–0.0422	–0.0230	–0.0067
$K\theta_a$	0.0961	0.0886	0.0678	0.0393	0.0124
K_{MC}	0.2062	0.1754	0.1197	0.0621	0.0177

$$Q_a = \frac{-w}{2\,a}\,(a^2 - r_o^2);\quad \text{If } r_o = 0,\quad G_{11} = 0.015,\quad G_{14} = 0.062,\quad G_{17} = \frac{(3+\upsilon)}{16}$$

$$L\,T_\delta = \frac{-w\,a^4}{D}\,G_{11};\quad L\,T_\theta = \frac{-w\,a^3}{D}\,G_{14};\quad \delta_C = \frac{-w\,a^4\,(5+\upsilon)}{64\,D\,(1+\upsilon)};\quad M_C = \frac{w\,a^2\,(3+\upsilon)}{16};\quad \theta_a = \frac{w\,a^3}{8\,D\,(1+\upsilon)}$$

Linear Increase Load; Edge Simply Supported



$$\delta_a = 0,\quad M_{ra} = 0;\quad M_C = w\,a^2\,G_{18}$$

$$\delta_C = \frac{-w\,a^4}{2\,D}\left(\frac{G_{18}}{1+\upsilon} - 2\,G_{12}\right)$$

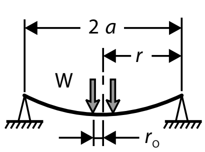
$$Q_a = \frac{-w}{6\,a}\,(2\,a^2 - r_o\,a - r_o^2)$$

r_o/a	0.00	0.20	0.40	0.60	0.80
$K\delta_C$	–0.0323	–0.0249	–0.0164	–0.0083	–0.0023
$K\theta_a$	0.0512	0.0407	0.0278	0.0148	0.0043
K_{MC}	0.0955	0.0708	0.0449	0.0222	0.0061

$$\theta_a = \frac{w\,a^3}{D}\left(\frac{G_{18}}{1+\upsilon} - 2\,G_{15}\right);\quad \text{If } r_o = 0,\quad G_{12} = 0.004,\quad G_{15} = 0.022,\quad G_{18} = \frac{(4+\upsilon)}{45}$$

$$L\,T_\delta = \frac{-w\,a^5 - r_o}{D\,a - r_o}\,G_{12};\quad \delta_C = \frac{-w\,a^4\,(6+\upsilon)}{15\,D\,(1+\upsilon)};\quad M_C = \frac{w\,a^2\,(4+\upsilon)}{45};\quad \theta_a = \frac{w\,a^3}{15\,D\,(1+\upsilon)}$$

Central Circular Load; Edge Simply Supported



$$\text{For } r > r_o;\quad \delta = \frac{W}{16\,\pi\,D}\left[\frac{(3+\upsilon)}{(1+\upsilon)}\,(a^2 - r^2) - 2\,r^2\,\ln\frac{a}{r}\right];\quad \theta = \frac{W\,r}{4\,\pi\,D}\left[\frac{1}{(1+\upsilon)} + \ln\frac{a}{r}\right]$$

$$M_r = \frac{W}{16\,\pi}\left[4\,(1+\upsilon)\ln\frac{a}{r} + (1-\upsilon)\,\frac{(a^2 - r^2)}{a^2}\,\frac{N^2}{r^2}\right];\quad N = \sqrt{1.6\,r_o^2 + t^2} - 0.67\,t;\quad \text{If } r_o < 0.5\,t$$

$$\text{or } N = r_o,\quad \text{If } r_o > 0.5\,t;\quad M_t = \frac{W}{16\,\pi}\left[4\,(1+\upsilon)\ln\frac{a}{r} + (1-\upsilon)\left(4 - \frac{N^2}{r^2}\right)\right]$$

$$\text{at } r = a;\quad \delta_{Max.} = \frac{-W\,a^2\,(3+\upsilon)}{16\,\pi\,D\,(1+\upsilon)};\quad \theta_{Max.} = \frac{W\,a}{4\,\pi\,D\,(1+\upsilon)};\quad M_{Max.} = \frac{W}{4\,\pi}\left[(1+\upsilon)\ln\frac{a}{N} + 1\right]$$

Besides the usual loadings, Table 2.5 Part II also includes several loading cases that may be described best as externally applied conditions that force a lack of flatness into the flat circular plates.

The first time we look at Table 2.5, Part II it appears to be a formidable task to calculate the strength of these structures. However, when we consider the number of cases it is possible to present in a limited space, the reason for this method of presentation becomes clear. With careful inspection, we find that the constants and functions with subscripts are the same except for the change in variables. In Table 2.5, Part II, the tabulated values in the Special Cases are listed for the preceding functions for the most frequently used denominator values of the variable ratios, such as b / a and r_0 / a.

Example 2-41

A flat circular plate is made of nylon 6/6 with 33% fiber glass reinforcement at 73 °F and 50% relative humidity. The radius is 3.00 in with a wall thickness of 0.25 in. The plate is simply supported around its edge and it is loaded with 500.00 lb at the center. The load is distributed through a round area of 0.125 in radius. Determine the maximum bending stress at the surface of the plate and the maximum deflection at the center of the plate.

Solution

This flat circular plate and loading are covered in Table 2.5, Part I, case load at center with the outer edge simply supported. The diagram and equations in Figure 2-96 were obtained from this table:

$t = 0.250$ in, $w = 500$ lb, $a = 3.00$ in, $r_0 = 0.125$ in,

$E = 900{,}00$ psi, $\upsilon = 0.39$, $\sigma = 18{,}000$ psi

$$\sigma_{\text{Max.}} = \frac{3\,W\,(1+v)}{2\,\pi\,t^2}\left(\frac{1}{v+1} + \log\frac{a}{r_0} - \frac{1 - v\,r_0^2}{1 + v\,4\,a^2}\right)$$

$$= \frac{3 \times 500\,(1+0.39)}{2 \times 3.1416 \times 0.25^2}\left(\frac{1}{0.39+1} + \log\frac{3.00}{0.125} - \frac{1 - 0.39 \times 0.125^2}{1 + 0.39 \times 4 \times 3.0^2}\right)$$

$= 10{,}794$ psi

$$\delta_{\text{Max.}} = \frac{3\,W\,a^2\,(1-v)\,(3+v)}{4\,\pi\,E\,t^3} = \frac{3 \times 500 \times 3.0^2\,(1-0.39)\,(3+0.39)}{4 \times 3.1416 \times 900{,}00 \times 0.25^3}$$

$= 0.158$ in

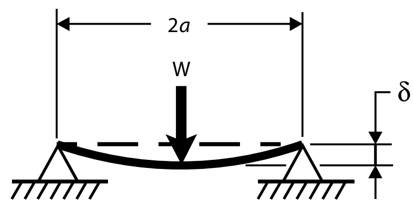


Figure 2-96 Flat circular plate, concentrated center load, and simply supported edge

Example 2-42

A thick flat circular plate is made of nylon 6/6 with 33% fiber glass reinforcement at 73 °F and 50% relative humidity with a radius of 4.00 in and a uniform wall thickness of 0.50 in. The plate's outer edge is fixed and it is uniformly loaded along the round area of plate with 200.00 lb/in.

Determine the maximum bending stress at the surface of the plate and the maximum deflection at the center of the plate.

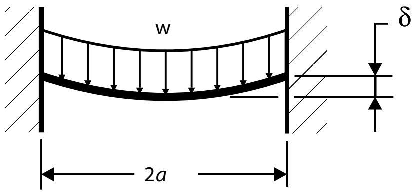


Figure 2-97 Flat circular plate, uniformly distributed load, and fixed edge

Solution

This thick flat circular plate and type of loading is presented in Table 2.5, Part I, case Uniformly Distributed Load with the Outer Edge Fixed. The diagram and equations in Figure 2-97 were obtained from the table.

Because this example case deals with a thick plate, we need to investigate if the thickness / radius ratio is greater than 0.1 to modify the maximum deflection by multiplying the value by the constant (H).

$t = 0.500$ in, $w = 200$ psi, $a = 4.00$ in, $E = 900{,}00$ psi,

$\upsilon = 0.39, \sigma = 18{,}000$ psi

For thicker flat circular plates with a ratio $t / a > 0.1$, multiply the deflection equation by the constant (H), where $H = 1 + 5.72\,(t / a)^2$.

$$\frac{t}{a} = \frac{0.50}{4.00} = 0.125 > 0.1$$

$$H = 1 + 5.72\,(t/a)^2 = 1 + 5.72\,(0.50/4.00)^2 = 1.089$$

$$\sigma_{\text{Max.}} = \frac{3\,w\,a^2}{\pi\,t^2} = \frac{3 \times 200 \times 4.0^2}{3.1416 \times 0.50^2} = 12{,}223.00 \text{ psi}$$

$$\delta = \frac{3\,w\,a^4\,(1 - \upsilon^2)}{16\,E\,t^3} = \frac{3 \times 200 \times 4.0^4\,(1 - 0.39^2)}{16 \times 900{,}00 \times 0.50^3} = 0.072 \text{ in}$$

$$\delta_{\text{Max.}} = H\ x\ y = 1.089 \times 0.072 = 0.079 \text{ in}$$

Example 2-43

A flat circular plate, made of acetal homopolymer, has a wall thickness of 0.187 in and a 5.00 in outside diameter, and is simply supported with a uniformly distributed load of 6.0 psi. Calculate the maximum deflection in the center, the maximum stress, and the deflection equation for Figure 2-98.

This flat circular plate and type of loading is presented in Table 2.5, Part II, case Uniformly Distributed Load Edge Simply Supported. First, we need to determine the maximum moment, the bending stress, the plate constant, and the deflection caused by the load. Second, we need to calculate the total deflection of the plate caused by the load, the moment, and the loading constant. Finally, we need to check the deflection at the outer edge.

$t = 0.187$ in, $w = 6.0$ psi, $a = 2.50$ in, $r_0 = 0$,

$E = 410{,}000$ psi, $\upsilon = 0.35, \sigma = 10{,}000$ psi

$$M_{\text{Max.}} = M_{\text{Center}} = \frac{w\,a^2\,(3 + \upsilon)}{16} = \frac{6.0 \times 2.50^2\,(3 + 0.35)}{16} = 7.85 \text{ lb-in.}$$

$$\sigma_{\text{Max.}} = \frac{6\,M}{t^2} = \frac{6 \times 7.85}{0.187^2} = 1{,}339.97 \text{ psi}$$

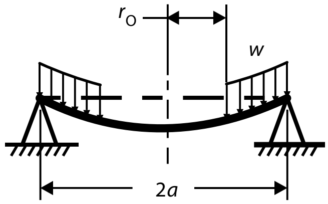


Figure 2-98 Flat circular plate, uniformly distributed load with simply supported edge

$$D = \frac{E\,t^3}{12\,(1-\upsilon^2)} = \frac{410{,}000 \times 0.187^3}{12\,(1-0.35^2)} = 256.66$$

$$\delta_C = \frac{-w\,a^4\,(5+\upsilon)}{64\,D\,(1+\upsilon)} = \frac{-6.0 \times 2.50^4\,(5+0.35)}{64 \times 256.66\,(1+0.35)} = -0.0565 \text{ in}$$

The total deflection equation for the flat circular plate is:

$$\delta_a = \delta_C + \frac{M_C\,y^2}{2\,D\,(1+\upsilon)} + L\,T_y\text{, where for this case } L\,T_y = \frac{-w\,a^4}{D}\,G_{11}$$

Where the constant $G_{11} = 0.015$, when $r_0 = 0$.

$$\delta_a = -0.0565 + \frac{7.85 \times a^2}{2 \times 256.66 \times 1.35} - \frac{6.0 \times 0.015 \times a^4}{256.66}$$
$$= -0.0565 + 0.01132 \times a^2 - 0.000365 \times a^4$$

Checking the deflection at the outer edge, when $a = 2.50$ in

$$\delta_a = -0.0565 + 0.01132 \cdot 2.50^2 - 0.000365 \cdot 2.50^4$$
$$= -0.0565 + 0.07075 - 0.01425 = 0.0$$

Example 2-44

A flat circular plate is made of acetal homopolymer with a wall thickness of 0.125 in and 4.00 in outside diameter. It is mounted in a fixture to produce a sudden change in slope in the radial direction of 0.05 radiant at a radius of 0.75 in. It is then clamped between two flat fixtures as shown in Figure 2-99.

Calculate the maximum bending stress.

This is an example of forcing a known change in slope into a flat circular plate, clamped (fixed) at both inner and outer edges. This flat circular plate and type of loading is presented in Table 2.5, Part II, case Outer and Inner Edge Fixed and Change in Slope, where: $\theta_0 = 0.05$, $b\,/\,a = 0.10$, $r_0\,/\,a = 0.50$ and Poisson's ratio of $\upsilon = 0.35$.

$t = 0.125$ in, $a = 1.50$ in, $b = 0.15$ in, $r_0 = 0.75$ in,

$\theta_0 = 0.05$ rad., $\theta_b = 0.0$ rad., $\delta_b = 0.0$ in, $E = 410{,}000$ psi,

$\upsilon = 0.35$, $\sigma = 10{,}000$ psi

$$D = \frac{E\,t^3}{12\,(1-\upsilon^2)} = \frac{410{,}000 \times 0.125^3}{12\,(1-0.35^2)} = 76.04$$

$$M_{rb} = \frac{K_{Mrb} \times \theta \times D}{a} = \frac{-2.054 \times 0.05 \times 76.04}{1.50} = -5.20 \text{ lb-in.}$$

$$Q_b = \frac{K_{Qb} \times \theta \times D}{a^2} = \frac{-0.0915 \times 0.05 \times 76.04}{1.50^2} = -0.154 \text{ lb/in.}$$

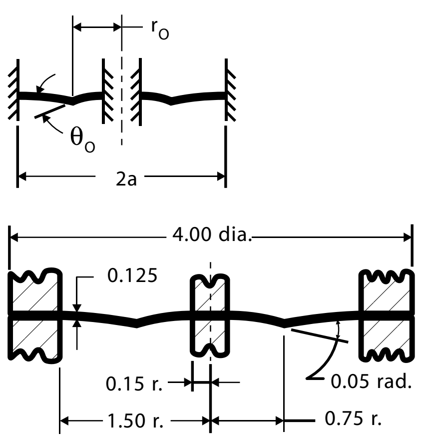


Figure 2-99 Flat circular plate having a change in slope with both outer and inner edges fixed

$$M_{ra} = M_{rb}\, C_8 + Q_b\, a\, C_9 + \theta_0 \frac{D}{a} L_7$$

$$= -5.20 \times C_8 + (-0.154) \times a \times C_9 + \frac{0.05 \times 76.04}{a} L_7$$

$$\sigma_{Max.} = \frac{6 \times M_{rb}}{t^2} = \frac{6 \times 5.20}{0.125^2} = 1{,}996.80 \text{ psi}$$

$$\delta_{Max.} = K\, y_0\, \theta\, r_0 = -0.1071 \times 0.05 \times 1.50 = 0.008 \text{ in}$$

Example 2-45

A flat circular plate, made of acetal homopolymer, has a wall thickness of 0.250 in and 5.00 in outside diameter, it is simply supported at the outer edge and subjected to two types of loads. One center load provides a uniform pressure over a diameter of 0.0625 in. The other is axis-symmetrically loaded with a distributed load that increases linearly from the center to the outside radius r_O = 1.00 in;, this load has a value of 10.00 psi at the outer edge.

Calculate the maximum bending stress.

This example requires analyzing two different cases and to superposition the results. The first case is the linear increase of the distributed load with simply outer edge supported (Figure 2-100), the second case is the central circular uniform load with simply supported outer edge (Figure 2-101). Both cases are presented in Table 2.5, Part II.

t = 0.250 in, a = 2.50 in, r_{01} = 1.00 in, r_{02} = 0.031 in,

E = 410,000 psi, υ = 0.35, σ = 10,000 psi

From the special case data, the following variable ratios are obtained:

$r_{01} / a = 1 / 2.5 = 0.40$, $K\, y_C = -0.0164$, $K\, \theta_a = 0.0278$, $K_{MC} = 0.0449$

$$D = \frac{E\, t^3}{12\,(1 - \upsilon^2)} = \frac{410{,}000 \times 0.250^3}{12\,(1 - 0.35^2)} = 608.38$$

$$\delta = K\, y_C \frac{w\, a^4}{D} = \frac{-0.0164 \times 10 \times 2.5^4}{608.38} = 0.0105 \text{ in}$$

$$M = K_{MC}\, w\, a^2 = 0.0449 \times 10 \times 2.5^2 = 2.80 \text{ lb-in.}$$

$$\delta_{Max.} = \frac{-P\, a^2\,(3 + \upsilon)}{16\, \pi\, D\,(1 + \upsilon)} = \frac{-P \times 2.50^2\,(3 + 0.35)}{16 \times \pi \times 608.38\,(1 + 0.35)} = 0.0105 \text{ in}$$

$$P = -20.76 \text{ lb.}$$

The second moment component is calculated by using the equations provided in Table 2.5, Part II, case Central Circular Loading and Simply Outer Edge Supported.

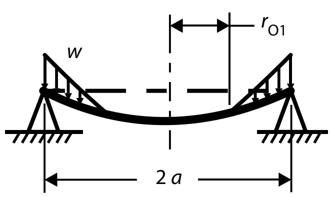


Figure 2-100 First case: Linear decreasing distributed load and edge simply supported

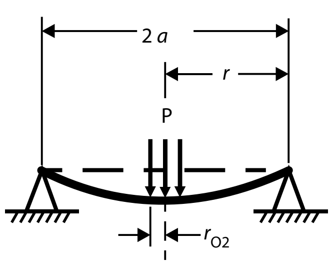


Figure 2-101 Second case: Center uniformly circular load and edge simply supported

$$N = \sqrt{1.6\, r_{02}^2 + t^2} - 0.675\, t = \sqrt{1.6 \times 0.03^2 + 0.25^2} - 0.675 \times 0.25$$
$$= 0.085 \text{ in}$$

$$M_{\text{Max.}} = \frac{P}{4\,\pi}\left[(1 + v)\ln\frac{a}{N} + 1\right] = \frac{-20.76}{4\,\pi}\left[(1 + 0.35)\ln\frac{2.50}{0.085} + 1\right]$$
$$= -9.19 \text{ lb-in.}$$

$$\sigma_{\text{Max.}} = \frac{6\,M}{t^2} = \frac{6\,(-9.19 + 2.80)}{0.250^2} = -613.44 \text{ psi}$$

2.17 Torsion Structural Analysis

A bar is rigidly clamped at one end and twisted at the other end by a torque $T = F \times d$, applied in a plane perpendicular to the axis. Plane sections remain plane and radii remain straight. There is at any point a shear stress (τ) on the plane of the section; the magnitude of this stress is proportional to the distance from the center of the section and its direction is perpendicular to the radius drawn through the point. The deformation and stresses are shown in Figure 2-102.

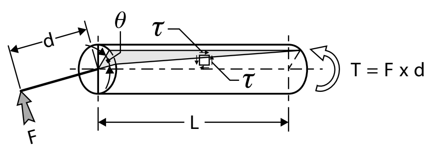


Figure 2-102 Deformation and stress under torque

In addition to these deformations and shear stresses, there are the longitudinal strain and stress. The longitudinal strain is reduced while the stress is in tension on the outside and a balancing compression stress is exerted on the inside.

Assumptions

The torsion equations are based on the following assumptions:

- The bar is straight, of uniform circular cross section (solid or tubing), and of homogeneous isotropic material.
- The bar is loaded only by equal and opposite twisting couples, which are applied at its ends in a normal direction to its axis.
- The bar is not stressed beyond the elastic limit of the material.

Angle of Twist (θ)

If a shaft of length (L) is subjected to a constant twisting moment (T) along its length, then θ is the angle through which only one end of the bar will be twisted.

Twisting Moment (T)

The twisting moment T for any section along the bar is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section in question.

Shearing Strain

If a bar is marked on the surface (unloaded), then after the twisting moment (T) has been applied, this line moves as shown in Figure 2-102. The angle (θ) is measured in radians; the final and original position of the generator is defined as the shearing strain at the surface of the bar.

Shearing Stress (τ)

For a solid circular cross section bar, let T = Twisting moment; L = Length of the bar; r_0 = Radius; J = Polar moment of inertia; τ = Shear stress; θ = Angle of twist (radians); G = Modulus of rigidity. Then:

$$\theta = (T\,L)/(G\,J)\,; \quad \tau_{\text{Max.}} = (T\,r_0)/J$$

By substituting for $J = (\pi\,r_0^4)/2$ in the equation above for a solid circular cross section with radius r_0, the following equations are obtained:

$$\theta = (2\,T\,L)/(\pi\,r_0^4\,G)\,; \quad \tau_{\text{Max.}} = (2\,T)/(\pi\,r_0^3)$$

For a circular tube cross section with outer radius r_0 and inner radius r_i:

$$\theta = (2\,T\,L)/\pi\,(r_0^4 - r_i^4)\,G]\,; \quad \tau_{\text{Max.}} = (2\,T\,r_0)/[\pi\,(r_0^4 - r_i^4)]$$

The torsional stiffness of the bar can be expressed by the general equation: $\theta = (T\,L)\,/\,(G\,K)$, where K is a factor dependent on the bar cross section. For cross section bars, the factor K is equivalent to the polar moment of inertia J. In Table 2-6, the equations for the factor K and for the maximum shear stress ($\tau_{\text{Max.}}$) for a variety of cross section bars are given.

Example 2-46

Compare the strength and stiffness of a circular injection molded tube made of a plastic material, 1.00 in outside diameter and 0.187 in wall thickness, versus an extruded solid circular bar of the same material with the same diameter.

The strengths of both cross sections will be compared by using the twisting moments (T) required to produce the same shear stress. The stiffness will be compared by using the values of factor (K) for both cross sections.

For the circular tube bar:

$$K = \pi\,(r_0^4 - r_i^4)/2 = 3.1416\,(0.50^4 - 0.313^4)/2 = 0.083\text{ in}^4$$

$$T = \tau\,\pi\,(r_0^4 - r_i^4)/(2\,r_0) = \tau \times 3.1416\,(0.50^4 - 0.313^4)/(2 \times 0.50)$$
$$= \tau \times 0.166\text{ lb-in.}$$

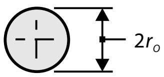
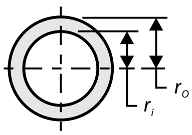
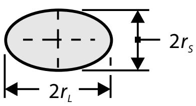
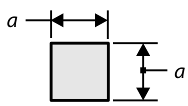
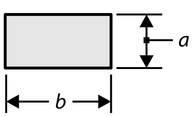
For the solid circular bar:

$$K = \pi\,r^4/2 = 3.1416 \times 0.50^4/2 = 0.098\text{ in}^4$$

$$T = (\tau\,\pi\,r^3)/2 = \tau \times 3.1416 \times 0.50^3/2 = \tau \times 0.196\text{ lb-in.}$$

The solid circular cross section bar is therefore 1.182 times as stiff as the circular tube cross section bar and 1.18 times as strong.

Table 2-6 Torsion Equations

Cross section	Constant K in $\theta = \dfrac{T\,L}{K\,G}$	Shear stress max.
Solid circle 	$K = \dfrac{\pi\, r_O^4}{2}$	$\tau_{Max.} = \dfrac{2\,T}{\pi\, r_O^3}$
Circular tube 	$K = \dfrac{\pi\,(r_O^4 - r_i^4)}{2}$	$\tau_{Max.} = \dfrac{2\,T\,r_O}{\pi\,(r_O^4 - r_i^4)}$
Solid ellipse 	$K = \dfrac{\pi\, r_L^3\, r_S^3}{r_L^2 + r_S^2}$	$\tau_{Max.} = \dfrac{2\,T}{\pi\, r_L\, r_S^2}$
Solid square 	$K = 0.1406\, a^4$	$\tau_{Max.} = \dfrac{T}{0.208\, a^3}$
Solid rectangle 	$K = \dfrac{b\,a^3}{16}\left[5.33 - 3.36\,\dfrac{a}{b}\left(1 - \dfrac{a^4}{12\,b^4}\right)\right]$	$\tau_{Max.} = \dfrac{T\,(3\,b + 1.8\,a)}{b^2\, a^2}$

θ = Angle of twist (radians); T = Twisting moment (lb-in); τ = Shear stress (psi); G = Modulus of rigidity (psi); J = Polar moment of inertia (in^4); K = Constant equivalent to J (in^4); r_o = Outer radius (in); r_i = Inner radius (in); r_S = Elliptical short radius (in); r_L = Elliptical large radius (in); a = Height (in); b = Width (in).

3 Structural Designs for Thermoplastics

3.1 Uniform and Symmetrical Wall Thickness

The ultimate design rule for injection molding thermoplastic products is to ensure that the wall thickness is uniform and symmetrical.

Non-uniform and/or heavy wall thicknesses can cause serious warpage and dimensional control problems in the injection molded products. Heavy wall sections cause not only internal shrinkage, voids, and surface sink marks, but also nonuniform shrinkage resulting in poor dimensional control and warpage problems.

Figure 3-1 shows a poor cross section design of perpendicular corner walls that causes molding problems, such as differential shrinkage, warpage (concave) of both walls, and internal voids in the corner of the thicker wall. The last two designs are recommended to avoid these molding problems.

Figure 3-2 shows a heavy wall cross section design that could cause molding problems and the recommended design using a thin wall and proportional ribs.

Figure 3-3 shows a nonuniform wall section that should be replaced with a thin uniform wall having the same strength of the original heavy wall section.

Figure 3-4 shows another poor and the recommended uniform wall design.

Figures 3-5 and 3-6 show cross sections of two nonuniform wall designs and the recommended designs with a uniform wall thickness to avoid warpage, internal voids, long molding cycles, and surface sink marks.

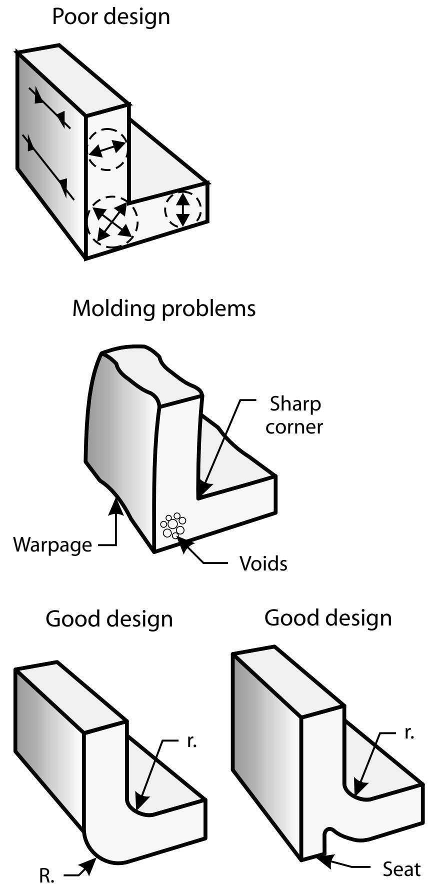


Figure 3-1 Perpendicular walls, end corner designs

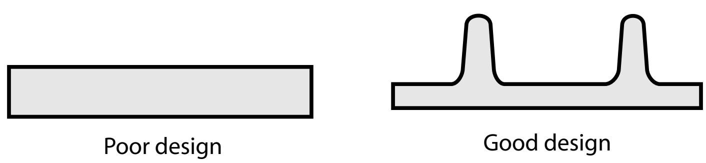


Figure 3-2 Heavy wall vs. thin uniform ribbed wall designs

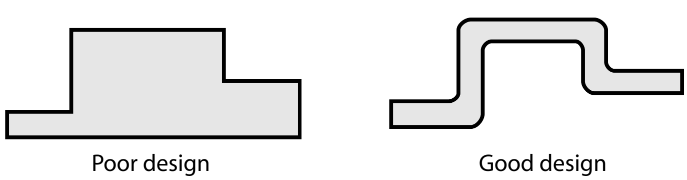


Figure 3-3 Nonuniform wall vs. thin uniform wall designs

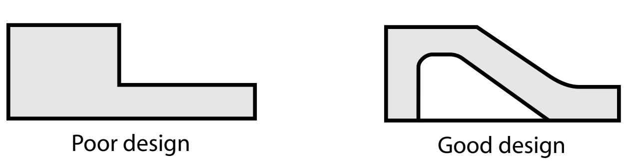


Figure 3-4 Nonuniform wall vs. thin uniform wall designs

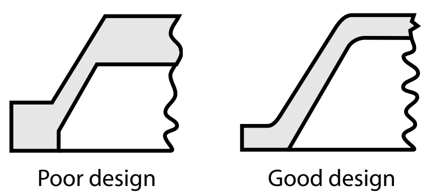


Figure 3-5 Nonuniform wall vs. thin uniform wall designs

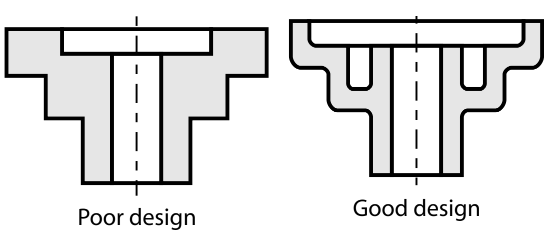


Figure 3-6 Nonuniform wall vs. thin uniform wall designs